

8 Goniometrische functies

Voorkennis Exacte waarden van goniometrische verhoudingen

Bladzijde 132

- 1 a** $2 \sin(45^\circ) + \sqrt{6} \cdot \sin(60^\circ) = 2 \cdot \frac{1}{2}\sqrt{2} + \sqrt{6} \cdot \frac{1}{2}\sqrt{3} = \sqrt{2} + \frac{1}{2}\sqrt{18} = \sqrt{2} + \frac{1}{2} \cdot 3\sqrt{2} = \sqrt{2} + 1\frac{1}{2}\sqrt{2} = 2\frac{1}{2}\sqrt{2}$
- b** $4 \sin(30^\circ) \tan(30^\circ) + 2 \cos(30^\circ) \cos(60^\circ) = 4 \cdot \frac{1}{2} \cdot \frac{1}{3}\sqrt{3} + 2 \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2} = \frac{2}{3}\sqrt{3} + \frac{1}{2}\sqrt{3} = \frac{4}{6}\sqrt{3} + \frac{3}{6}\sqrt{3} = 1\frac{1}{6}\sqrt{3}$
- 2 a** $4 \cos(30^\circ) + 9 \tan(30^\circ) = 4 \cdot \frac{1}{2}\sqrt{3} + 9 \cdot \frac{1}{3}\sqrt{3} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$
- b** $3 \sin(30^\circ) - \sqrt{2} \cdot \sin(45^\circ) = 3 \cdot \frac{1}{2} - \sqrt{2} \cdot \frac{1}{2}\sqrt{2} = 1\frac{1}{2} - 1 = \frac{1}{2}$
- c** $6 \tan(30^\circ) - 3 \tan(60^\circ) = 6 \cdot \frac{1}{3}\sqrt{3} - 3 \cdot \sqrt{3} = 2\sqrt{3} - 3\sqrt{3} = -\sqrt{3}$
- d** $\sqrt{2} \cdot \sin(60^\circ) + 3\sqrt{3} \cdot \sin(45^\circ) = \sqrt{2} \cdot \frac{1}{2}\sqrt{3} + 3\sqrt{3} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{6} + 1\frac{1}{2}\sqrt{6} = 2\sqrt{6}$
- 3 a** $8 \sin(45^\circ) \cos(30^\circ) = 8 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} = 2\sqrt{6}$
- b** $2\sqrt{3} \cdot \sin(60^\circ) \cos(30^\circ) = 2\sqrt{3} \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$
- c** $\sqrt{2} \cdot \sin(45^\circ) \sin(60^\circ) - 2 \tan(60^\circ) = \sqrt{2} \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} - 2 \cdot \sqrt{3} = \frac{1}{2}\sqrt{3} - 2\sqrt{3} = -1\frac{1}{2}\sqrt{3}$
- d** $(\sin(30^\circ) + \cos(30^\circ))^2 = (\frac{1}{2} + \frac{1}{2}\sqrt{3})^2 = \frac{1}{4} + \frac{1}{2}\sqrt{3} + \frac{3}{4} = 1 + \frac{1}{2}\sqrt{3}$
- 4 a** $\frac{\cos(30^\circ)}{1 + \sin(60^\circ)} = \frac{\frac{1}{2}\sqrt{3}}{1 + \frac{1}{2}\sqrt{3}} = \frac{\sqrt{3}}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2\sqrt{3} - 3}{1} = 2\sqrt{3} - 3$
- b** $\frac{\sin(30^\circ) + \sin(60^\circ)}{\sin(30^\circ) - \sin(60^\circ)} = \frac{\frac{1}{2} + \frac{1}{2}\sqrt{3}}{\frac{1}{2} - \frac{1}{2}\sqrt{3}} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{1 + 2\sqrt{3} + 3}{-2} = -2 - \sqrt{3}$
- 5 a** $\sin(15^\circ) = \sin(60^\circ - 45^\circ) = \sin(60^\circ) \cos(45^\circ) - \cos(60^\circ) \sin(45^\circ) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}$
- b** $\sin(75^\circ) = \sin(45^\circ + 30^\circ) = \sin(45^\circ) \cos(30^\circ) + \cos(45^\circ) \sin(30^\circ) = \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{2} \cdot \frac{1}{2} = \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}$

8.1 Eenheidscirkel en radiaal

Bladzijde 133

- 1 a** $\sin(\alpha) = \frac{PQ}{OP} = \frac{PQ}{1} = PQ$, dus $PQ = \sin(65^\circ) \approx 0,91$.
- $\cos(\alpha) = \frac{OQ}{OP} = \frac{OQ}{1} = OQ$, dus $OQ = \cos(65^\circ) \approx 0,42$.
- b** $P(0,42; 0,91)$
- c** $\angle POQ = 180^\circ - 115^\circ = 65^\circ$
 $PQ \approx 0,91$, $OQ \approx 0,42$ en $P(-0,42; 0,91)$.
- d** $\cos(115^\circ) \approx -0,42$ en $\sin(115^\circ) \approx 0,91$
 Dezelfde y-coördinaat en tegengestelde x-coördinaat, oftewel $x_P = \cos(115^\circ)$ en $y_P = \sin(115^\circ)$.
- e** $\angle POQ = 245^\circ - 180^\circ = 65^\circ$
 $P(-0,42; -0,91)$
 De GR geeft $\cos(245^\circ) \approx -0,42$ en $\sin(245^\circ) \approx -0,91$, oftewel $x_P = \cos(245^\circ)$ en $y_P = \sin(245^\circ)$.

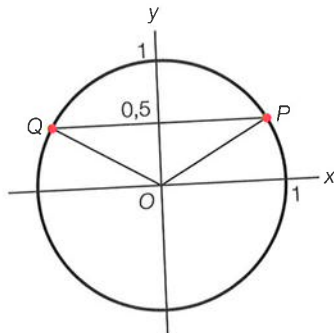
Bladzijde 134

- 2 a $\sin(0^\circ) = 0$
 b $\cos(0^\circ) = 1$
 c $\sin(90^\circ) = 1$
 d $\cos(90^\circ) = 0$
 e $\sin(270^\circ) = -1$
 f $\cos(270^\circ) = 0$
- g $\sin(360^\circ) = 0$
 h $\tan(360^\circ) = 0$
 i $\sin(450^\circ) = 1$
 j $\cos(-90^\circ) = 0$
 k $\tan(-360^\circ) = 0$
 l $\cos(-180^\circ) = -1$
- 3 a $\alpha = 0^\circ, \alpha = 180^\circ$ en $\alpha = 360^\circ$
 b $\alpha = 90^\circ$ en $\alpha = 270^\circ$
 c $\alpha = 90^\circ$
- d $\alpha = 0^\circ$ en $\alpha = 360^\circ$
 e $\alpha = 45^\circ$ en $\alpha = 225^\circ$
 f $\alpha = 135^\circ$ en $\alpha = 315^\circ$

Bladzijde 135

- 4 a *
- b $P(\cos(110^\circ), \sin(110^\circ)) \approx P(-0,34; 0,94)$
 $Q(\cos(200^\circ), \sin(200^\circ)) \approx Q(-0,94; -0,34)$
 $R(\cos(-102^\circ), \sin(-102^\circ)) \approx R(-0,21; -0,98)$
 $S(\cos(-50^\circ), \sin(-50^\circ)) \approx S(0,64; -0,77)$
- 5 $\frac{360^\circ}{5} = 72^\circ$ $2 \cdot 72^\circ = 144^\circ$ $3 \cdot 72^\circ = 216^\circ$ $4 \cdot 72^\circ = 288^\circ$
- $B(2 \cos(72^\circ), 2 \sin(72^\circ)) \approx B(0,62; 1,90)$
 $C(2 \cos(144^\circ), 2 \sin(144^\circ)) \approx C(-1,62; 1,18)$
 $D(2 \cos(216^\circ), 2 \sin(216^\circ)) \approx D(-1,62; -1,18)$
 $E(2 \cos(288^\circ), 2 \sin(288^\circ)) \approx E(0,62; -1,90)$

6 a



b $\beta = 180^\circ - 30^\circ = 150^\circ$

Bladzijde 136

- 7 a $x_P = 0,81$, dus $\cos(\alpha) = 0,81$.
 De GR geeft $35,90...^\circ$.
 Dus $\alpha \approx 35,9^\circ$.
- b $y_P = 0,94$, dus $\sin(\alpha) = 0,94$.
 De GR geeft $70,05...^\circ$.
 Dus $\alpha = 180^\circ - 70,05...^\circ \approx 109,9^\circ$.
- c $x_P = 0,26$, dus $\cos(\alpha) = 0,26$.
 De GR geeft $74,92...^\circ$.
 Dus $\alpha \approx -74,9^\circ$.
- d $y_P = -0,22$, dus $\sin(\alpha) = -0,22$.
 De GR geeft $-12,70...^\circ$.
 Dus $\alpha = 180^\circ + 12,70...^\circ \approx 192,7^\circ$.
- 8 $y_P = 0,92$, dus $\sin(\alpha_P) = 0,92$.
 De GR geeft $66,92...^\circ$, dus $\alpha_P = 66,92...^\circ$.
 $x_Q = -0,87$, dus $\cos(\alpha_Q) = -0,87$.
 De GR geeft $150,45...^\circ$, dus $\alpha_Q = 360^\circ - 150,45...^\circ = 209,54...^\circ$.
 $\angle POQ = \alpha_Q - \alpha_P = 209,54...^\circ - 66,92...^\circ \approx 142,6^\circ$

9 Zie de figuur hiernaast.

$$x_S = 0,527, \text{ dus } \cos(\alpha_S) = 0,527.$$

De GR geeft $58,197\dots^\circ$, dus $\alpha_S = 58,197\dots^\circ$.

$$\cos(58,197\dots^\circ + 160^\circ) = -0,785\dots$$

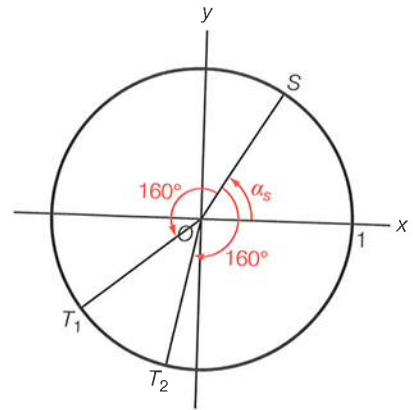
$$\sin(58,197\dots^\circ + 160^\circ) = -0,618\dots$$

Dus $T_1(-0,79; -0,62)$.

$$\cos(58,197\dots^\circ - 160^\circ) = -0,204\dots$$

$$\sin(58,197\dots^\circ - 160^\circ) = -0,978\dots$$

Dus $T_2(-0,20; -0,98)$.



Bladzijde 137

10 a omtrek cirkel $= 2\pi r$

In de eenheidscirkel is $r = 1$, dus omtrek $= 2\pi \cdot 1 = 2\pi$.

b Bij $\alpha = 90^\circ$ hoort een kwart cirkelboog, dus lengte cirkelboog $= \frac{1}{4} \cdot 2\pi = \frac{1}{2}\pi$.

draaiingshoek α	0°	90°	180°	270°	360°
lengte cirkelboog b	0	$\frac{1}{2}\pi$	π	$1\frac{1}{2}\pi$	2π

Bladzijde 138

11 a $\frac{1}{6}\pi \text{ rad} = \frac{1}{6} \cdot 180^\circ = 30^\circ$

b $\frac{1}{4}\pi \text{ rad} = \frac{1}{4} \cdot 180^\circ = 45^\circ$

c $2\pi \text{ rad} = 2 \cdot 180^\circ = 360^\circ$

d $2 \text{ rad} = 2 \cdot \frac{180^\circ}{\pi} \approx 114,6^\circ$

e $1\frac{1}{4}\pi \text{ rad} = 1\frac{1}{4} \cdot 180^\circ = 225^\circ$

f $1\frac{1}{4} \text{ rad} = 1\frac{1}{4} \cdot \frac{180^\circ}{\pi} \approx 71,6^\circ$

g $-2\frac{1}{3}\pi \text{ rad} = -2\frac{1}{3} \cdot 180^\circ = -420^\circ$

h $-2\frac{1}{3} \text{ rad} = -2\frac{1}{3} \cdot \frac{180^\circ}{\pi} \approx -133,7^\circ$

Bladzijde 139

12 a $360^\circ = 360 \cdot \frac{\pi}{180} \text{ rad} = 2\pi \text{ rad}$

b $30^\circ = 30 \cdot \frac{\pi}{180} \text{ rad} = \frac{1}{6}\pi \text{ rad}$

c $45^\circ = 45 \cdot \frac{\pi}{180} \text{ rad} = \frac{1}{4}\pi \text{ rad}$

d $60^\circ = 60 \cdot \frac{\pi}{180} \text{ rad} = \frac{1}{3}\pi \text{ rad}$

e $90^\circ = 90 \cdot \frac{\pi}{180} \text{ rad} = \frac{1}{2}\pi \text{ rad}$

f $135^\circ = 135 \cdot \frac{\pi}{180} \text{ rad} = \frac{3}{4}\pi \text{ rad}$

g $300^\circ = 300 \cdot \frac{\pi}{180} \text{ rad} = 1\frac{2}{3}\pi \text{ rad}$

h $210^\circ = 210 \cdot \frac{\pi}{180} \text{ rad} = 1\frac{1}{6}\pi \text{ rad}$

13 a $10^\circ = 10 \cdot \frac{\pi}{180} \text{ rad} \approx 0,17 \text{ rad}$

b $57,3^\circ = 57,3 \cdot \frac{\pi}{180} \text{ rad} \approx 1,00 \text{ rad}$

c $1030^\circ = 1030 \cdot \frac{\pi}{180} \text{ rad} \approx 17,98 \text{ rad}$

d $90^\circ = 90 \cdot \frac{\pi}{180} \text{ rad} \approx 1,57 \text{ rad}$

hoek in graden	40°	72°	100°	135°	145°	150°
hoek in radialen	$\frac{2}{9}\pi$	$\frac{2}{5}\pi$	$\frac{5}{9}\pi$	$\frac{3}{4}\pi$	$\frac{29}{36}\pi$	$\frac{5}{6}\pi$

15 a $\cos(\frac{5}{8}\pi) \approx -0,38$

b $\cos(\frac{5}{8}) \approx 0,81$

c $\sin(\frac{4}{5}\pi) \approx 0,59$

d $\sin(\frac{4}{5}) \approx 0,72$

e $\cos(7,6\pi) \approx 0,31$

f $\cos(7,6) \approx 0,25$

16 a $\alpha \approx 1,17$

b $\alpha \approx 0,55$

c $\alpha \approx 0,43$

d $\alpha \approx 1,39$

e $\alpha \approx 0,84$

f $\alpha \approx 1,21$

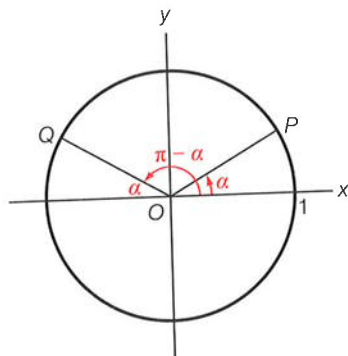
- 17 a** $\sin(\frac{1}{5}\pi) + \cos(\frac{2}{5}\pi) - \sin(\frac{3}{5}\pi) \approx 0,33$
b $\cos(2\alpha) = 0,6$ geeft $2\alpha = 0,927\dots$, dus $\alpha \approx 0,46$.

- 18 a** $y_P = 0,35$, dus $\sin(\alpha) = 0,35$.
 De GR geeft $0,357\dots$
 Dus $\alpha = \pi - 0,357\dots \approx 2,78$.
b $x_P = -0,35$, dus $\cos(\alpha) = -0,35$.
 De GR geeft $1,928\dots$
 Dus $\alpha = 2\pi - 1,928\dots \approx 4,35$.

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- 19** $x_P = -0,32$, dus $\cos(\alpha_P) = -0,32$.
 De GR geeft $1,896\dots$
 Dus $\alpha_P = 1,896\dots$
 $y_Q = -0,88$, dus $\sin(\alpha_Q) = -0,88$.
 De GR geeft $-1,075\dots$
 Dus $\alpha_Q = \pi + 1,075\dots = 4,217\dots$
 $\angle POQ = \alpha_Q - \alpha_P = 4,217\dots - 1,896\dots \approx 2,32$

20 a

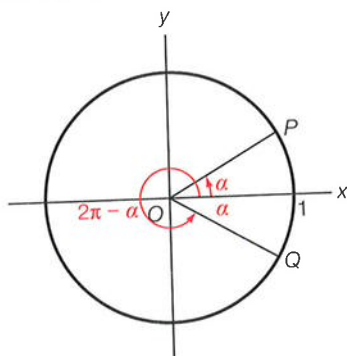


In de figuur zie je dat $y_Q = y_P$.

Dus $\sin(\pi - \alpha) = \sin(\alpha)$.

- b** In de figuur van vraag a zie je dat $x_Q = -x_P$.
 Dus $\cos(\pi - \alpha) = -\cos(\alpha)$.

c

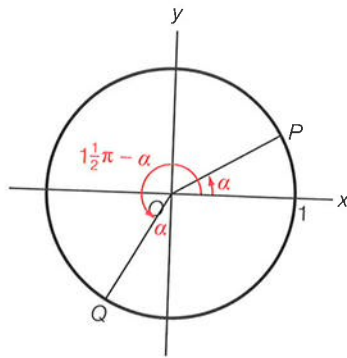


In de figuur zie je dat $y_Q = -y_P$.

Dus $\sin(2\pi - \alpha) = -\sin(\alpha)$.

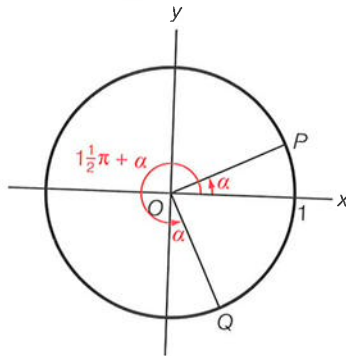
- d** In de figuur van vraag c zie je dat $x_Q = x_P$.
 Dus $\cos(2\pi - \alpha) = \cos(\alpha)$.

e



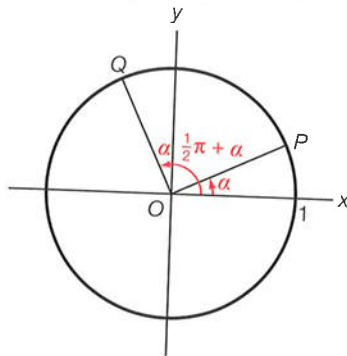
In de figuur zie je dat $y_Q = -x_P$.
Dus $\sin(\frac{1}{2}\pi - \alpha) = -\cos(\alpha)$.

f



In de figuur zie je dat $x_Q = y_P$.
Dus $\cos(\frac{1}{2}\pi + \alpha) = \sin(\alpha)$.

g



In de figuur zie je dat $y_Q = x_P$.
Dus $\sin(\frac{1}{2}\pi + \alpha) = \cos(\alpha)$.

h In de figuur van vraag g zie je dat $x_Q = -y_P$.
Dus $\cos(\frac{1}{2}\pi + \alpha) = -\sin(\alpha)$.

- 21 a $\frac{1}{6}\pi = \frac{1}{6} \cdot 180^\circ = 30^\circ$, dus $\cos(\frac{1}{6}\pi) = \cos(30^\circ) = \frac{1}{2}\sqrt{3}$.
b $\frac{1}{4}\pi = \frac{1}{4} \cdot 180^\circ = 45^\circ$, dus $\sin(\frac{1}{4}\pi) = \sin(45^\circ) = \frac{1}{2}\sqrt{2}$.

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- 22 a $\sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$ d $\cos(\frac{1}{3}\pi) = -\frac{1}{2}$
b $\cos(\frac{1}{6}\pi) = -\frac{1}{2}\sqrt{3}$ e $\cos(\frac{2}{3}\pi) = \frac{1}{2}$
c $\sin(\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$ f $\sin(-\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$

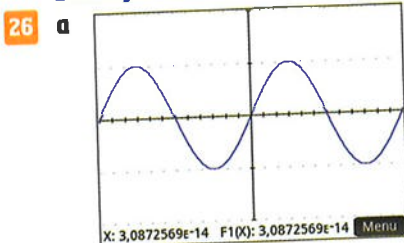
23 $\tan(\frac{5}{6}\pi) - \tan(\frac{2}{3}\pi) = \frac{\sin(\frac{5}{6}\pi)}{\cos(\frac{5}{6}\pi)} - \frac{\sin(\frac{2}{3}\pi)}{\cos(\frac{2}{3}\pi)} = \frac{-\frac{1}{2}}{\frac{1}{2}\sqrt{3}} - \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{-1}{\sqrt{3}} + \sqrt{3} = -\frac{1}{3}\sqrt{3} + \sqrt{3} = \frac{2}{3}\sqrt{3}$

- 24 a $\sin(\alpha) = \frac{1}{2}\sqrt{3}$ geeft $\alpha = \frac{1}{3}\pi \vee \alpha = \frac{2}{3}\pi$
 b $\cos(\alpha) = -\frac{1}{2}$ geeft $\alpha = \frac{2}{3}\pi \vee \alpha = \frac{4}{3}\pi$
 c $\sin(\alpha) = -\frac{1}{2}\sqrt{2}$ geeft $\alpha = \frac{5}{4}\pi \vee \alpha = \frac{7}{4}\pi$
 d $\cos(\alpha) = 0$ geeft $\alpha = \frac{1}{2}\pi \vee \alpha = \frac{3}{2}\pi$
 e $\cos(\alpha) = \frac{1}{2}\sqrt{3}$ geeft $\alpha = \frac{1}{6}\pi \vee \alpha = \frac{5}{6}\pi$
 f $\cos(\alpha) = \frac{1}{2}\sqrt{2}$ geeft $\alpha = \frac{1}{4}\pi \vee \alpha = \frac{7}{4}\pi$

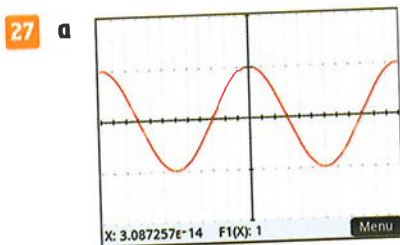
- 25 a $x = \sin(\frac{5}{6}\pi) = \frac{1}{2}$
 b $\sin(x) = -\frac{1}{2}\sqrt{3}$ met $0 \leq x \leq \frac{1}{2}\pi$ geeft $x = \frac{1}{3}\pi$
 c $x = \cos(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$
 d $\cos(x) = \frac{1}{2}$ met $\frac{1}{2}\pi \leq x \leq 2\pi$ geeft $x = \frac{2}{3}\pi$

8.2 Sinusoïden

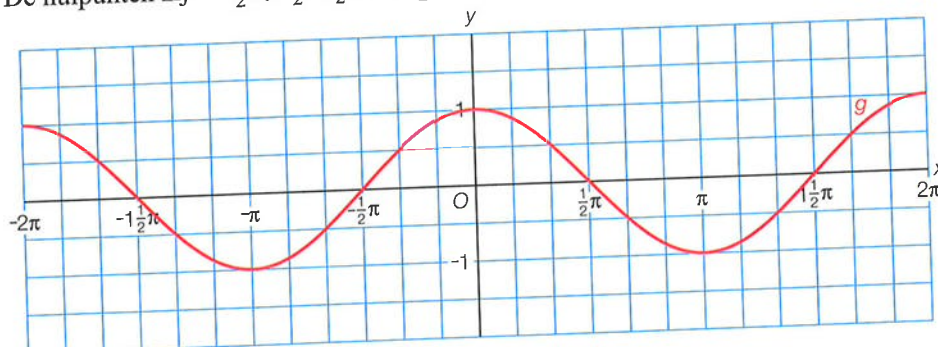
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- b De toppen zijn $(-\frac{1}{2}\pi, 1)$, $(\frac{1}{2}\pi, 1)$ en $(\frac{5}{2}\pi, 1)$.
 c De snijpunten met de x -as zijn $(-2\pi, 0)$, $(-\pi, 0)$, $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.



- b De toppen zijn $(-2\pi, 1)$, $(-\pi, -1)$, $(0, 1)$, $(\pi, -1)$ en $(2\pi, 1)$.
 c De nulpunten zijn $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$ en $\frac{3}{2}\pi$.
 d



Bladzijde 144

- 28 a** $f(x) = \sin(x)$
 $\downarrow$ translatie $(0, 2)$
 $g(x) = 2 + \sin(x)$
 evenwichtsstand = 2
- b** $f(x) = \sin(x)$
 $\downarrow$ translatie $(\frac{1}{3}\pi, 0)$
 $h(x) = \sin(x - \frac{1}{3}\pi)$
 De nulpunten van h zijn ..., $-2\frac{2}{3}\pi$, $-1\frac{2}{3}\pi$, $-\frac{2}{3}\pi$, $\frac{1}{3}\pi$, $1\frac{1}{3}\pi$, $2\frac{1}{3}\pi$, ...
- c** $f(x) = \sin(x)$
 $\downarrow$ verm. x -as, 4
 $k(x) = 4 \sin(x)$
 amplitude = 4
- d** $f(x) = \sin(x)$
 $\downarrow$ verm. y -as, $\frac{1}{5}$
 $l(x) = \sin(5x)$
 periode = $\frac{2\pi}{5} = \frac{2}{5}\pi$

Bladzijde 145

- 29 a** Ja, ik ben het met José eens.
b $(\frac{1}{3}\pi, 1\frac{1}{2})$

Bladzijde 146

- 30** Effect van transformaties op de standaardgrafiek $y = \cos(x)$

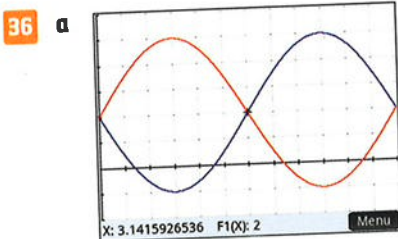
transformatie	beeldgrafiek	kenmerk
translatie $(0, a)$	$y = a + \cos(x)$	evenwichtsstand a
verm. x -as, b	$y = b \cos(x)$	amplitude b ($b > 0$)
verm. y -as, $\frac{1}{c}$	$y = \cos(cx)$	periode $\frac{2\pi}{c}$ ($c > 0$)
translatie $(d, 0)$	$y = \cos(x - d)$	beginpunt $(d, 1)$

- 31 a** De evenwichtsstand is 0, de amplitude is 2, de periode is 2π en een beginpunt is $(-3, 0)$.
b De evenwichtsstand is $\frac{1}{5}$, de amplitude is $\frac{1}{3}$, de periode is 2π en een beginpunt is $(0, \frac{1}{5})$.
c De evenwichtsstand is 0, de amplitude is 1, de periode is $\frac{2}{3}\pi$ en een beginpunt is $(4, 1)$.
d De evenwichtsstand is 0, de amplitude is $1\frac{1}{2}$, de periode is 8π en een beginpunt is $(0, 1\frac{1}{2})$.
- 32 a** De evenwichtsstand is 5, de amplitude is 1,2, de periode is 2π en een beginpunt is $(\frac{1}{6}\pi; 6,2)$.
b De evenwichtsstand is 0,4, de amplitude is 1, de periode is 10π en een beginpunt is $(-\frac{1}{3}\pi; 0,4)$.
c De evenwichtsstand is 0, de amplitude is 0,29, de periode is $\frac{2}{3}\pi$ en een beginpunt is $(-1,4; 0,29)$.
d De evenwichtsstand is -0,8, de amplitude is 2, de periode is $\frac{2}{3}\pi$ en een beginpunt is $(\frac{1}{2}\pi; -0,8)$.
- 33 a** $y = \cos(x)$
 $\downarrow$ translatie $(\frac{1}{4}\pi, 4)$
 $y = 4 + \cos(x - \frac{1}{4}\pi)$
 $\downarrow$ verm. x -as, 3
 $y = 3(4 + \cos(x - \frac{1}{4}\pi))$
 oftewel $y = 12 + 3 \cos(x - \frac{1}{4}\pi)$
 Dus $f(x) = 12 + 3 \cos(x - \frac{1}{4}\pi)$.

b $y = \cos(x)$
 $\downarrow$ verm. x -as, 3
 $y = 3 \cos(x)$
 $\downarrow$ translatie $(\frac{1}{4}\pi, 4)$
 $y = 4 + 3 \cos(x - \frac{1}{4}\pi)$
Dus $g(x) = 4 + 3 \cos(x - \frac{1}{4}\pi)$.

- 34 a** De evenwichtsstand is $-\frac{1}{2}$, de amplitude is 1, de periode is 2π en een beginpunt is $(\frac{1}{4}\pi, -\frac{1}{2})$.
b De grafiek van f snijdt de lijn van de evenwichtsstand in de punten $(\frac{1}{4}\pi, -\frac{1}{2})$, $(1\frac{1}{4}\pi, -\frac{1}{2})$ en $(2\frac{1}{4}\pi, -\frac{1}{2})$.
c De toppen van de grafiek van f zijn $(\frac{3}{4}\pi, \frac{1}{2})$, $(1\frac{3}{4}\pi, \frac{1}{2})$ en $(2\frac{3}{4}\pi, \frac{1}{2})$.
d De afstand tussen A en C is gelijk aan de periode, dus $AC = 2\pi$.

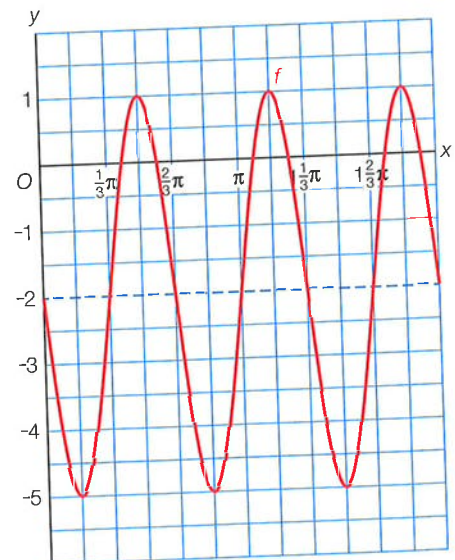
35 $f(x) = 3 + 4 \sin(x - \frac{1}{2}\pi)$
 $\downarrow$ translatie $(-\frac{3}{4}\pi, -5)$
 $y = -5 + 3 + 4 \sin(x + \frac{3}{4}\pi - \frac{1}{2}\pi)$
oftewel $y = -2 + 4 \sin(x + \frac{1}{4}\pi)$
Dus $m = -\frac{3}{4}\pi$ en $n = -5$ zijn mogelijke waarden van m en n .



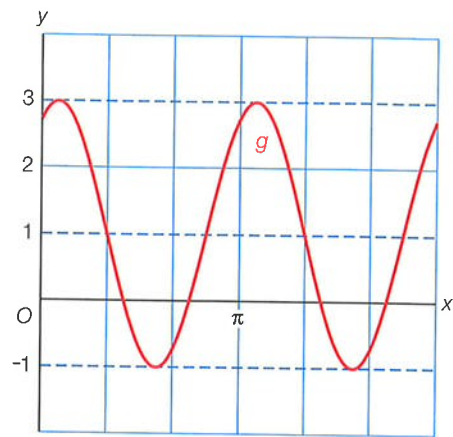
- b** Van de grafiek van $f(x) = 2 + 3 \sin(x)$ is de amplitude 3 en van de grafiek van $g(x) = 2 - 3 \sin(x)$ is de amplitude 3.

Bladzijde 149

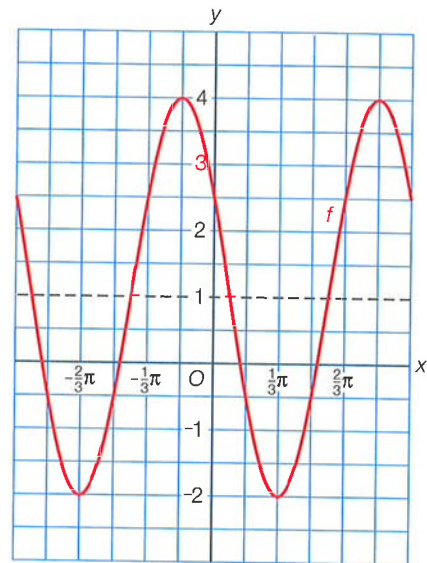
- 37 a** $f(x) = -2 + 3 \sin(3x + \pi) = -2 + 3 \sin(3(x + \frac{1}{3}\pi))$
evenwichtsstand -2
amplitude 3
periode $\frac{2\pi}{3} = \frac{2}{3}\pi$
 $3 > 0$, dus grafiek stijgend door het punt $(-\frac{1}{3}\pi, -2)$ en dus ook stijgend door het punt $(\frac{1}{3}\pi, -2)$.



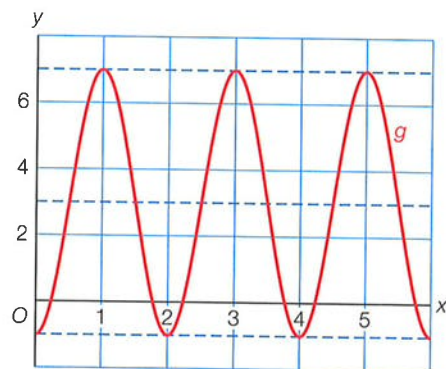
- b** $g(x) = 1 - 2 \sin(2x - \frac{2}{3}\pi) = 1 - 2 \sin(2(x - \frac{1}{3}\pi))$
 evenwichtsstand 1
 amplitude 2
 periode $\frac{2\pi}{2} = \pi$
 $-2 < 0$, dus grafiek dalend door het punt $(\frac{1}{3}\pi, 1)$.



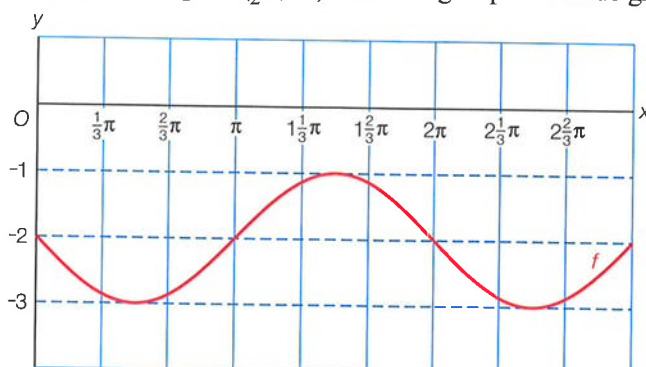
- 38 a** $f(x) = 1 + 3 \cos(2x + \frac{1}{3}\pi) = 1 + 3 \cos(2(x + \frac{1}{6}\pi))$
 evenwichtsstand 1
 amplitude 3
 periode $\frac{2\pi}{2} = \pi$
 $3 > 0$, dus $(-\frac{1}{6}\pi, 4)$ is een hoogste punt van de grafiek.



- b** $g(x) = 3 - 4 \cos(\pi x)$
 evenwichtsstand 3
 amplitude 4
 periode $\frac{2\pi}{\pi} = 2$
 $-4 < 0$, dus het punt $(0, -1)$ is een laagste punt van de grafiek.



- 39 a** $f(x) = -2 - \cos(x - \frac{1}{2}\pi)$
 evenwichtsstand -2
 amplitude 1
 periode 2π
 $-1 < 0$, dus het punt $(\frac{1}{2}\pi, -3)$ is een laagste punt van de grafiek.



b $g(x) = 5 - 3 \sin(\frac{1}{4}\pi x)$

evenwichtsstand 5

amplitude 3

periode $\frac{2\pi}{\frac{1}{4}\pi} = 8$

$-3 < 0$, dus de grafiek gaat dalend door het punt $(0, 5)$.



40 $y = 2 - 3 \sin(\frac{1}{2}x + \frac{1}{6}\pi)$ oftewel $y = 2 - 3 \sin(\frac{1}{2}(x + \frac{1}{3}\pi))$

evenwichtsstand 2

amplitude 3

Dalend door de evenwichtsstand bij $x = -\frac{1}{3}\pi$.

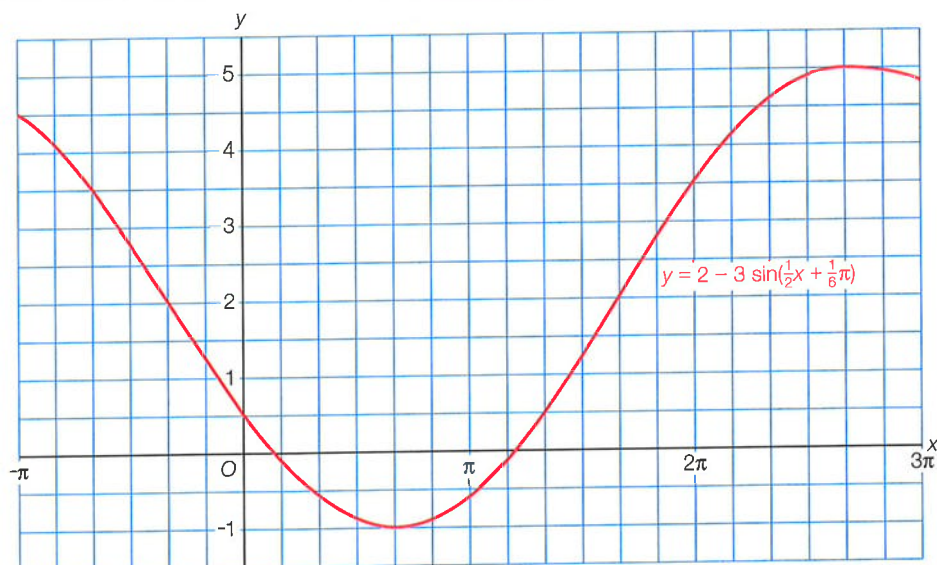
periode $\frac{2\pi}{\frac{1}{2}} = 4\pi$

Tussen het laagste en het hoogste punt zitten horizontaal 12 roosterhokjes, dus $\frac{1}{2}$ periode $= 2\pi$ komt overeen met 12 roosterhokjes, dus horizontaal komt 1 roosterhokje overeen met $\frac{1}{6}\pi$.

Tussen het laagste en het hoogste punt zitten verticaal 12 roosterhokjes, dus $2 \cdot \text{amplitude} = 6$ komt overeen met 12 roosterhokjes, dus verticaal komt 1 roosterhokje overeen met $\frac{1}{2}$.

Het laagste punt is $(-\frac{1}{3}\pi + \frac{1}{4} \cdot 4\pi, 2 - 3) = (\frac{2}{3}\pi, -1)$.

Teken de verticale as 4 roosterhokjes links van het laagste punt en teken de horizontale as 2 roosterhokjes boven het laagste punt. Je krijgt de figuur hieronder.



41 **a** evenwichtsstand 3

amplitude 2

periode π

$$\begin{aligned} \text{b } a &= 3 \\ b &= 2 \\ c &= \frac{2\pi}{\pi} = 2 \\ d &= \frac{1}{3}\pi \end{aligned}$$

Bladzijde 150

42 a De grafiek gaat dalend door de evenwichtsstand bij $x = \pi$, dus $y = 1 - 2\frac{1}{2} \sin(1\frac{1}{2}(x - \pi))$.

b Het punt $(0, -1\frac{1}{2})$ is een laagste punt, dus $y = 1 - 2\frac{1}{2} \cos(1\frac{1}{2}x)$.

43 a De grafiek gaat dalend door de evenwichtsstand bij $x = 3$, dus $y = 1 - 2\frac{1}{2} \sin(\frac{1}{2}\pi(x - 3))$.

b Het punt $(0, -1\frac{1}{2})$ is een laagste punt, dus $y = 1 - 2\frac{1}{2} \cos(\frac{1}{2}\pi x)$.

44 a $a = \frac{3\frac{1}{2} + \frac{1}{2}}{2} = 2$

$$b = 3\frac{1}{2} - 2 = 1\frac{1}{2}$$

$$\frac{1}{2} \text{ periode} = \pi - \frac{1}{2}\pi = \frac{1}{2}\pi, \text{ dus periode} = \pi, \text{ dus } c = \frac{2\pi}{\pi} = 2.$$

$$\text{Stijgend door de evenwichtsstand bij } x = \frac{1}{2}\pi, \text{ dus } d = \frac{1}{2}\pi.$$

$$\text{Dus } y = 2 + 1\frac{1}{2} \sin(2(x - \frac{1}{2}\pi)).$$

b Een hoogste punt is $(\frac{3}{4}\pi, 3\frac{1}{2})$, dus $y = 2 + 1\frac{1}{2} \cos(2(x - \frac{3}{4}\pi))$.

Bladzijde 151

45 a $a = \frac{4 + -2}{2} = 1$

$$b = 4 - 1 = 3$$

$$\frac{1}{2} \text{ periode} = 2\frac{3}{4} - 1\frac{1}{4} = 1\frac{1}{2}, \text{ dus periode} = 3, \text{ dus } c = \frac{2\pi}{3} = \frac{2}{3}\pi.$$

$$\text{Stijgend door de evenwichtsstand bij } x = \frac{1\frac{1}{4} + 2\frac{3}{4}}{2} = 2, \text{ dus } d = 2.$$

$$\text{Dus } y = 1 + 3 \sin(\frac{2}{3}\pi(x - 2)).$$

b De grafiek gaat dalend door de evenwichtsstand bij $x = 2 - \frac{1}{2} \cdot \text{periode} = 2 - \frac{1}{2} \cdot 3 = \frac{1}{2}$, dus $y = 1 - 3 \sin(\frac{2}{3}\pi(x - \frac{1}{2}))$.

46 a $a = \frac{175 + 25}{2} = 100$

$$b = 175 - 100 = 75$$

$$\frac{1}{2} \text{ periode} = 9 - 4 = 5, \text{ dus periode} = 10, \text{ dus } c = \frac{2\pi}{10} = \frac{1}{5}\pi.$$

$$\text{Stijgend door de evenwichtsstand bij } t = 4, \text{ dus } d = 4.$$

$$\text{Dus } N = 100 + 75 \sin(\frac{1}{5}\pi(t - 4)).$$

b Er is een hoogste punt bij $t = 4 + \frac{1}{4} \cdot \text{periode} = 4 + \frac{1}{4} \cdot 10 = 6\frac{1}{2}$.

$$\text{Dus } N = 100 + 75 \cos(\frac{1}{5}\pi(t - 6\frac{1}{2})).$$

47 a Stel $y = a + b \cos(c(x - d))$.

$$a = 12 - 4 = 8$$

$$b = 4$$

$$\text{De periode is } 18 - 3 = 15, \text{ dus } c = \frac{2\pi}{15} = \frac{2}{15}\pi.$$

$$\text{Het punt } (3, 12) \text{ is een hoogste punt, dus } d = 3.$$

$$\text{Dus } y = 8 + 4 \cos(\frac{2}{15}\pi(x - 3)).$$

- b** Stel $y = a + b \cos(c(x - d))$.

$$a = 650$$

$$b = 812 - 650 = 162$$

$$\text{De periode is 48, dus } c = \frac{2\pi}{48} = \frac{1}{24}\pi.$$

Het punt (16, 812) is een hoogste punt, dus $d = 16$.

$$\text{Dus } y = 650 + 162 \cos\left(\frac{1}{24}\pi(x - 16)\right).$$

- c** Stel $y = a + b \cos(c(x - d))$.

$$a = \frac{250 + 110}{2} = 180$$

$$b = 250 - 180 = 70$$

$$\frac{1}{2} \text{ periode} = 26 - 2 = 24, \text{ dus periode} = 48, \text{ dus } c = \frac{2\pi}{48} = \frac{1}{24}\pi.$$

Het punt (2, 250) is een hoogste punt, dus $d = 2$.

$$\text{Dus } y = 180 + 70 \cos\left(\frac{1}{24}\pi(x - 2)\right).$$

8.3 Goniometrische vergelijkingen

Bladzijde 153

48 ..., $-3\frac{1}{2}\pi$, $-2\frac{1}{2}\pi$, $-1\frac{1}{2}\pi$, $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$, $1\frac{1}{2}\pi$, $2\frac{1}{2}\pi$, $3\frac{1}{2}\pi$, ...

Bladzijde 154

49 a $\sin(3x - \frac{1}{2}\pi) = 0$

$$3x - \frac{1}{2}\pi = k \cdot \pi$$

$$3x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$$

b $\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0$

$$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$$

$$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$$

$$x = 1\frac{1}{3}\pi + k \cdot 2\pi$$

c $\sin^2(x) - \sin(x) = 0$

$$\sin(x)(\sin(x) - 1) = 0$$

$$\sin(x) = 0 \vee \sin(x) = 1$$

$$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi$$

d $\cos^2(2x) + \cos(2x) = 0$

$$\cos(2x)(\cos(2x) + 1) = 0$$

$$\cos(2x) = 0 \vee \cos(2x) = -1$$

$$2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = \pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = \frac{1}{2}\pi + k \cdot \pi$$

Bladzijde 155

50 a $\cos^2(x - \frac{1}{5}\pi) = 1$

$$\cos(x - \frac{1}{5}\pi) = 1 \vee \cos(x - \frac{1}{5}\pi) = -1$$

$$x - \frac{1}{5}\pi = k \cdot 2\pi \vee x - \frac{1}{5}\pi = \pi + k \cdot 2\pi$$

$$x = \frac{1}{5}\pi + k \cdot 2\pi \vee x = 1\frac{1}{5}\pi + k \cdot 2\pi$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = \frac{1}{5}\pi + k \cdot \pi$.

b $\sin^2(2x - \frac{1}{4}\pi) = 1$

$$\sin(2x - \frac{1}{4}\pi) = 1 \vee \sin(2x - \frac{1}{4}\pi) = -1$$

$$2x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$2x = \frac{3}{4}\pi + k \cdot 2\pi \vee 2x = 1\frac{3}{4}\pi + k \cdot 2\pi$$

$$x = \frac{3}{8}\pi + k \cdot \pi \vee x = \frac{7}{8}\pi + k \cdot \pi$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi$.

c $\sin^3(x) - \sin(x) = 0$

$$\sin(x)(\sin^2(x) - 1) = 0$$

$$\sin(x) = 0 \vee \sin^2(x) = 1$$

$$\sin(x) = 0 \vee \sin(x) = 1 \vee \sin(x) = -1$$

$$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = 1\frac{1}{2}\pi + k \cdot 2\pi$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot \frac{1}{2}\pi$.

$$\begin{aligned}
 \text{d } \cos^3(2x) - \cos(2x) &= 0 \\
 \cos(2x)(\cos^2(2x) - 1) &= 0 \\
 \cos(2x) = 0 \vee \cos^2(2x) = 1 \\
 \cos(2x) = 0 \vee \cos(2x) = 1 \vee \cos(2x) = -1 \\
 2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = k \cdot 2\pi \vee 2x = \pi + k \cdot 2\pi \\
 x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot \pi
 \end{aligned}$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot \frac{1}{4}\pi$.

$$\begin{aligned}
 \text{51 a } \sin(4x - \frac{1}{3}\pi) &= 1 \\
 4x - \frac{1}{3}\pi &= \frac{1}{2}\pi + k \cdot 2\pi \\
 4x &= \frac{5}{6}\pi + k \cdot 2\pi \\
 x &= \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \cos(4\pi x) &= -1 \\
 4\pi x &= \pi + k \cdot 2\pi \\
 x &= \frac{1}{4} + k \cdot \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \sin^2(\frac{1}{4}\pi x) &= 1 \\
 \sin(\frac{1}{4}\pi x) &= 1 \vee \sin(\frac{1}{4}\pi x) = -1 \\
 \frac{1}{4}\pi x &= \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{4}\pi x = 1\frac{1}{2}\pi + k \cdot 2\pi \\
 x &= 2 + k \cdot 8 \vee x = 6 + k \cdot 8
 \end{aligned}$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = 2 + k \cdot 4$.

$$\begin{aligned}
 \text{d } \sin(2x) \cos(2x) + \sin(2x) &= 0 \\
 \sin(2x)(\cos(2x) + 1) &= 0 \\
 \sin(2x) = 0 \vee \cos(2x) = -1 \\
 2x = k \cdot \pi \vee 2x = \pi + k \cdot 2\pi \\
 x = k \cdot \frac{1}{2}\pi \vee x = \frac{1}{2}\pi + k \cdot \pi
 \end{aligned}$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot \frac{1}{2}\pi$.

$$\begin{aligned}
 \text{52 a } 2 \sin(x) \cos(2x) &= 0 \\
 \sin(x) = 0 \vee \cos(2x) &= 0 \\
 x = k \cdot \pi \vee 2x = \frac{1}{2}\pi + k \cdot \pi \\
 x = k \cdot \pi \vee x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi
 \end{aligned}$$

Het domein van f is $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{4}\pi \vee x = \frac{3}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi$.

De nulpunten zijn $0, \frac{1}{4}\pi, \frac{3}{4}\pi, \pi, 1\frac{1}{4}\pi, 1\frac{3}{4}\pi$ en 2π .

$$\begin{aligned}
 \text{b } rc_{AB} &= \frac{2 - -2}{1\frac{1}{2}\pi - \frac{1}{2}\pi} = \frac{4}{\pi} \\
 y = \frac{4}{\pi}x + b &\left\{ \begin{array}{l} \frac{4}{\pi} \cdot \frac{1}{2}\pi + b = -2 \\ 2 + b = -2 \\ b = -4 \end{array} \right.
 \end{aligned}$$

$$\text{Dus } AB: y = \frac{4}{\pi}x - 4.$$

$$\text{Snijden met de } x\text{-as geeft } \frac{4}{\pi}x - 4 = 0$$

$$\frac{4}{\pi}x = 4$$

$$x = \pi$$

Dus de lijn door A en B snijdt de x -as in $(\pi, 0)$, een van de snijpunten van de grafiek van f met de x -as.

- c $f(\frac{1}{6}\pi) = 2 \sin(\frac{1}{6}\pi) \cdot \cos(\frac{1}{3}\pi) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$, dus C ligt op de grafiek van f .
 $f(\frac{1}{6}\pi) = 2 \sin(\frac{1}{6}\pi) \cdot \cos(\frac{2}{3}\pi) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$, dus D ligt op de grafiek van f .

d $rc_{CD} = \frac{-\frac{1}{2} - \frac{1}{2}}{\frac{1}{6}\pi - \frac{1}{6}\pi} = \frac{-1}{\pi} = -\frac{1}{\pi}$

$$y = -\frac{1}{\pi}x + b \quad \left\{ \begin{array}{l} -\frac{1}{\pi} \cdot \frac{1}{6}\pi + b = \frac{1}{2} \\ -\frac{1}{6} + b = \frac{1}{2} \\ b = \frac{2}{3} \end{array} \right.$$

Dus $CD: y = -\frac{1}{\pi}x + \frac{2}{3}$.

Snijden met de x -as geeft $-\frac{1}{\pi}x + \frac{2}{3} = 0$

$$-\frac{1}{\pi}x = -\frac{2}{3}$$

$$x = \frac{2}{3}\pi$$

Dus de lijn door C en D snijdt de x -as in $(\frac{2}{3}\pi, 0)$, dat is geen snijpunt van de grafiek van f met de x -as.

- 53 a $x = \frac{1}{6}\pi$ geeft $\sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = \frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
 b $x = 2\frac{1}{6}\pi$ geeft $\sin(2\frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = 2\frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
 $x = 4\frac{1}{6}\pi$ geeft $\sin(4\frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = 4\frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
 c $x = \frac{5}{6}\pi$ geeft $\sin(\frac{5}{6}\pi) = \frac{1}{2}$, dus $x = \frac{5}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
 d $x = 2\frac{5}{6}\pi$ geeft $\sin(2\frac{5}{6}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$, dus $x = 2\frac{5}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
 $x = -1\frac{1}{6}\pi$ geeft $\sin(-1\frac{1}{6}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$, dus $x = -1\frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

Bladzijde 157

- 54 a $2 \sin(\frac{1}{2}x) = 1$
 $\sin(\frac{1}{2}x) = \frac{1}{2}$
 $\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot 4\pi \vee x = 1\frac{2}{3}\pi + k \cdot 4\pi$
- b $2 \cos(x - \frac{1}{3}\pi) = 1$
 $\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$
 $x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi$
- c $2 \sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$
 $\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$
 $2x - \frac{1}{4}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{4}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$
 $2x = 1\frac{7}{12}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{12}\pi + k \cdot 2\pi$
 $x = \frac{19}{24}\pi + k \cdot \pi \vee x = -\frac{1}{24}\pi + k \cdot \pi$
- d $2 \cos(3x - \pi) = -1$
 $\cos(3x - \pi) = -\frac{1}{2}$
 $3x - \pi = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x - \pi = -\frac{2}{3}\pi + k \cdot 2\pi$
 $3x = 1\frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

55 a $2 \sin(2x - \frac{1}{6}\pi) = \sqrt{2}$
 $\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$
 $2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$
 $2x = \frac{5}{12}\pi + k \cdot 2\pi \vee 2x = \frac{11}{12}\pi + k \cdot 2\pi$
 $x = \frac{5}{24}\pi + k \cdot \pi \vee x = \frac{11}{24}\pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = \frac{5}{24}\pi \vee x = \frac{11}{24}\pi \vee x = \frac{11}{24}\pi \vee x = \frac{11}{24}\pi$

b $2 \cos(3x - \frac{1}{2}\pi) = \sqrt{3}$
 $\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$
 $3x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$
 $3x = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{2}{9}\pi \vee x = \frac{8}{9}\pi \vee x = \frac{5}{9}\pi \vee x = \frac{1}{9}\pi \vee x = \frac{7}{9}\pi \vee x = \frac{4}{9}\pi$

c $\sin(\frac{2}{3}x) = \frac{1}{2}\sqrt{2}$
 $\frac{2}{3}x = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{2}{3}x = \frac{3}{4}\pi + k \cdot 2\pi$
 $x = \frac{3}{8}\pi + k \cdot 3\pi \vee x = \frac{1}{8}\pi + k \cdot 3\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{3}{8}\pi \vee x = \frac{1}{8}\pi$

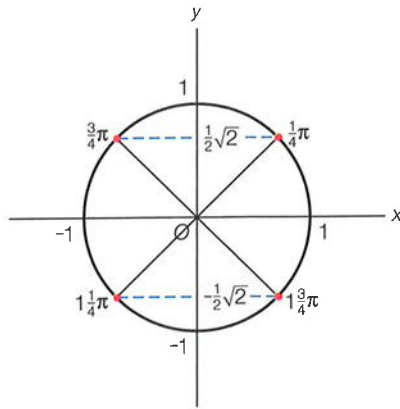
d $\cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{3}$
 $\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{3}\pi + k \cdot 4\pi \vee x = -\frac{5}{3}\pi + k \cdot 4\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{5}{3}\pi$

56 a $2 \sin^2(x) = 1$
 $\sin^2(x) = \frac{1}{2}$
 $\sin(x) = \sqrt{\frac{1}{2}} \vee \sin(x) = -\sqrt{\frac{1}{2}}$
 $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$
Dus uit $2 \sin^2(x) = 1$ volgt $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$.

b Uit $\sin(x) = \frac{1}{2}\sqrt{2}$ volgt $x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi$.
Uit $\sin(x) = -\frac{1}{2}\sqrt{2}$ volgt $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{1}{4}\pi + k \cdot 2\pi$.
Dus uit $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$ volgt
 $x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{1}{4}\pi + k \cdot 2\pi$.

c $x = \frac{1}{4}\pi + k \cdot 2\pi : \dots, -1\frac{3}{4}\pi, \frac{1}{4}\pi, 2\frac{1}{4}\pi, \dots$
 $x = \frac{3}{4}\pi + k \cdot 2\pi : \dots, -1\frac{1}{4}\pi, \frac{3}{4}\pi, 2\frac{3}{4}\pi, \dots$
 $x = \frac{1}{4}\pi + k \cdot 2\pi : \dots, -\frac{3}{4}\pi, \frac{1}{4}\pi, 3\frac{1}{4}\pi, \dots$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi : \dots, -\frac{1}{4}\pi, \frac{3}{4}\pi, 3\frac{3}{4}\pi, \dots$
Dus $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.

d



Uit de cirkel is af te lezen dat $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$ geeft $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.

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57 a $2 \cos^2(\frac{1}{2}x) = 1$

$$\cos^2(\frac{1}{2}x) = \frac{1}{2}$$

$$\cos(\frac{1}{2}x) = \sqrt{\frac{1}{2}} \vee \cos(\frac{1}{2}x) = -\sqrt{\frac{1}{2}}$$

$$\cos(\frac{1}{2}x) = \frac{1}{2}\sqrt{2} \vee \cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{2}$$

$$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

$$x = \frac{1}{2}\pi + k \cdot \pi$$

b $4 \sin^2(x - \frac{1}{6}\pi) = 1$

$$\sin^2(x - \frac{1}{6}\pi) = \frac{1}{4}$$

$$\sin(x - \frac{1}{6}\pi) = \frac{1}{2} \vee \sin(x - \frac{1}{6}\pi) = -\frac{1}{2}$$

$$x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \pi + k \cdot 2\pi \vee x = k \cdot 2\pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot

$$x = \frac{1}{3}\pi + k \cdot \pi \vee x = k \cdot \pi.$$

c $4 \cos^2(x + \frac{1}{4}\pi) = 3$

$$\cos^2(x + \frac{1}{4}\pi) = \frac{3}{4}$$

$$\cos(x + \frac{1}{4}\pi) = \sqrt{\frac{3}{4}} \vee \cos(x + \frac{1}{4}\pi) = -\sqrt{\frac{3}{4}}$$

$$\cos(x + \frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \vee \cos(x + \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$$

$$x + \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{12}\pi + k \cdot 2\pi \vee x = -\frac{5}{12}\pi + k \cdot 2\pi \vee x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = -\frac{13}{12}\pi + k \cdot 2\pi$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot

$$x = -\frac{1}{12}\pi + k \cdot \pi \vee x = -\frac{5}{12}\pi + k \cdot \pi.$$

d $4 \sin^3(x) - \sin(x) = 0$

$$\sin(x)(4 \sin^2(x) - 1) = 0$$

$$\sin(x) = 0 \vee 4 \sin^2(x) = 1$$

$$\sin(x) = 0 \vee \sin^2(x) = \frac{1}{4}$$

$$\sin(x) = 0 \vee \sin(x) = \frac{1}{2} \vee \sin(x) = -\frac{1}{2}$$

$$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot

$$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot \pi \vee x = \frac{5}{6}\pi + k \cdot \pi.$$

58 a $\sin(\frac{1}{2}\pi x) = \frac{1}{2}\sqrt{3}$

$$\frac{1}{2}\pi x = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$$

$$x = \frac{2}{3} + k \cdot 4 \vee x = \frac{1}{3} + k \cdot 4$$

$$x \text{ in } [0, 10] \text{ geeft } x = \frac{2}{3} \vee x = \frac{10}{3} \vee x = \frac{14}{3} \vee x = \frac{18}{3} \vee x = \frac{22}{3} \vee x = \frac{26}{3} \vee x = \frac{30}{3}$$

b $\cos(\frac{1}{3}\pi x) = -\frac{1}{2}\sqrt{3}$

$$\frac{1}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{3}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$x = 2\frac{1}{2} + k \cdot 6 \vee x = -2\frac{1}{2} + k \cdot 6$$

$$x \text{ in } [0, 10] \text{ geeft } x = 2\frac{1}{2} \vee x = 8\frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 9\frac{1}{2}$$

c $4\sin^2(\frac{1}{5}\pi x) = 1$

$$\sin^2(\frac{1}{5}\pi x) = \frac{1}{4}$$

$$\sin(\frac{1}{5}\pi x) = \frac{1}{2} \vee \sin(\frac{1}{5}\pi x) = -\frac{1}{2}$$

$$\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = -\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{6} + k \cdot 10 \vee x = 4\frac{1}{6} + k \cdot 10 \vee x = -\frac{5}{6} + k \cdot 10 \vee x = 5\frac{5}{6} + k \cdot 10$$

$$x \text{ in } [0, 10] \text{ geeft } x = \frac{5}{6} \vee x = 4\frac{1}{6} \vee x = 9\frac{1}{6} \vee x = 5\frac{5}{6}$$

d $2\cos^2(0,1\pi x) + \cos(0,1\pi x) = 0$

$$\cos(0,1\pi x)(2\cos(0,1\pi x) + 1) = 0$$

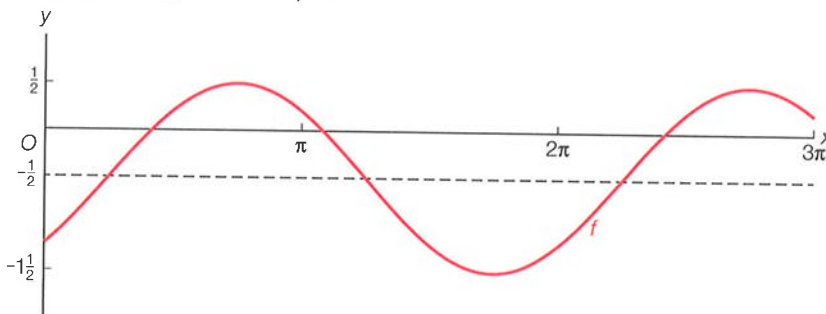
$$\cos(0,1\pi x) = 0 \vee \cos(0,1\pi x) = -\frac{1}{2}$$

$$0,1\pi x = \frac{1}{2}\pi + k \cdot \pi \vee 0,1\pi x = \frac{2}{3}\pi + k \cdot 2\pi \vee 0,1\pi x = -\frac{2}{3}\pi + k \cdot 2\pi$$

$$x = 5 + k \cdot 10 \vee x = 6\frac{2}{3} + k \cdot 20 \vee x = -6\frac{2}{3} + k \cdot 20$$

$$x \text{ in } [0, 10] \text{ geeft } x = 5 \vee x = 6\frac{2}{3}$$

59 a Voer in $y_1 = -\frac{1}{2} + \sin(x - \frac{1}{4}\pi)$.



b $f(x) = -\frac{1}{2}$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$

$$\sin(x - \frac{1}{4}\pi) = 0$$

$$x - \frac{1}{4}\pi = k \cdot \pi$$

$$x = \frac{1}{4}\pi + k \cdot \pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{1}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = 2\frac{1}{4}\pi$$

De grafiek van f snijdt de lijn van de evenwichtsstand in de punten

$$(\frac{1}{4}\pi, -\frac{1}{2}), (1\frac{1}{4}\pi, -\frac{1}{2}) \text{ en } (2\frac{1}{4}\pi, -\frac{1}{2}).$$

c $f(x) = \frac{1}{2}$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = \frac{1}{2}$

$$\sin(x - \frac{1}{4}\pi) = 1$$

$$x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{3}{4}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = 2\frac{3}{4}\pi$$

$$f(x) = -1\frac{1}{2} \text{ geeft } -\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1\frac{1}{2}$$

$$\sin(x - \frac{1}{4}\pi) = -1$$

$$x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$x = 1\frac{3}{4}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = 1\frac{3}{4}\pi$$

De toppen van de grafiek van f zijn $(\frac{3}{4}\pi, \frac{1}{2})$, $(1\frac{3}{4}\pi, -1\frac{1}{2})$ en $(2\frac{3}{4}\pi, \frac{1}{2})$.

d $f(x) = 0$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = 0$

$$\sin(x - \frac{1}{4}\pi) = \frac{1}{2}$$

$$x - \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{12}\pi + k \cdot 2\pi \vee x = 1\frac{1}{12}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{5}{12}\pi \vee x = 2\frac{5}{12}\pi \vee x = 1\frac{1}{12}\pi$$

$$\text{Dus } x_A = \frac{5}{12}\pi, x_B = 1\frac{1}{12}\pi \text{ en } x_C = 2\frac{5}{12}\pi.$$

$$AB = x_B - x_A = 1\frac{1}{12}\pi - \frac{5}{12}\pi = \frac{8}{12}\pi = \frac{2}{3}\pi$$

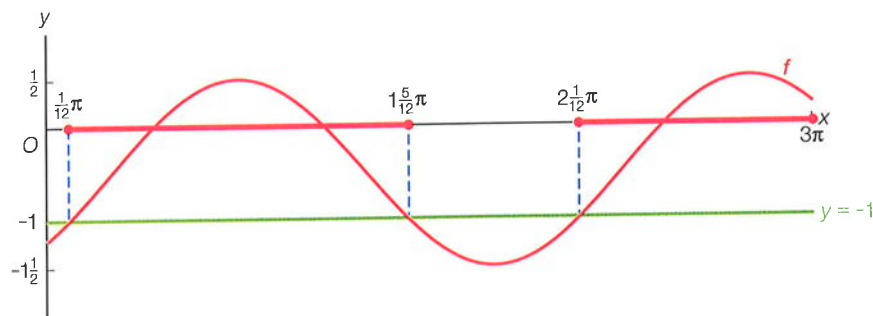
e $f(x) = -1$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1$

$$\sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$$

$$x - \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee x = 1\frac{5}{12}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{1}{12}\pi \vee x = 2\frac{1}{12}\pi \vee x = 1\frac{5}{12}\pi$$



$$f(x) \geq -1 \text{ geeft } \frac{1}{12}\pi \leq x \leq 1\frac{5}{12}\pi \vee 2\frac{1}{12}\pi \leq x \leq 3\pi$$

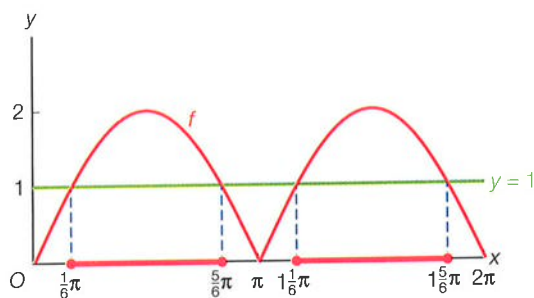
60 $f(x) = 1$ geeft $|2 \sin(x)| = 1$

$$2 \sin(x) = 1 \vee 2 \sin(x) = -1$$

$$\sin(x) = \frac{1}{2} \vee \sin(x) = -\frac{1}{2}$$

$$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$$



$$f(x) \geq 1 \text{ geeft } \frac{1}{6}\pi \leq x \leq \frac{5}{6}\pi \vee 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$$

61 a $\sin(3x) = \sin(\frac{1}{6}\pi)$

$$\sin(3x) = \frac{1}{2}$$

$$3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$$

b $\cos(3x) = \cos(\frac{1}{6}\pi)$

$$\cos(3x) = \frac{1}{2}\sqrt{3}$$

$$3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$$

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62

a $\sin(x+1) = \sin(2x+3)$

$$x+1 = 2x+3 + k \cdot 2\pi \vee x+1 = \pi - (2x+3) + k \cdot 2\pi$$

$$-x = 2 + k \cdot 2\pi \vee x+1 = \pi - 2x - 3 + k \cdot 2\pi$$

$$x = -2 + k \cdot 2\pi \vee 3x = -4 + \pi + k \cdot 2\pi$$

$$x = -2 + k \cdot 2\pi \vee x = -\frac{1}{3} + \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$$

b $\cos(2x-1) = \cos(x+1)$

$$2x-1 = x+1 + k \cdot 2\pi \vee 2x-1 = -(x+1) + k \cdot 2\pi$$

$$x = 2 + k \cdot 2\pi \vee 2x-1 = -x-1 + k \cdot 2\pi$$

$$x = 2 + k \cdot 2\pi \vee 3x = k \cdot 2\pi$$

$$x = 2 + k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$$

c $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$

$$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$$

$$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$$

d $\cos(x - \frac{1}{3}\pi) = \cos(2x)$

$$x - \frac{1}{3}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$$

$$-x = \frac{1}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

e $\sin(2\pi x) = \sin(\pi(x-1))$

$$\sin(2\pi x) = \sin(\pi x - \pi)$$

$$2\pi x = \pi x - \pi + k \cdot 2\pi \vee 2\pi x = \pi - (\pi x - \pi) + k \cdot 2\pi$$

$$\pi x = -\pi + k \cdot 2\pi \vee 2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$$

$$x = -1 + k \cdot 2 \vee 3\pi x = 2\pi + k \cdot 2\pi$$

$$x = -1 + k \cdot 2 \vee x = \frac{2}{3} + k \cdot \frac{2}{3}$$

$$x = -1 + k \cdot 2 \vee x = k \cdot \frac{2}{3}$$

f $\cos(\frac{1}{2}\pi x) = \cos(\pi(x-2))$

$$\cos(\frac{1}{2}\pi x) = \cos(\pi x - 2\pi)$$

$$\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = -(\pi x - 2\pi) + k \cdot 2\pi$$

$$-\frac{1}{2}\pi x = -2\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$$

$$x = 4 + k \cdot 4 \vee 1\frac{1}{2}\pi x = 2\pi + k \cdot 2\pi$$

$$x = k \cdot 4 \vee x = 1\frac{1}{3} + k \cdot 1\frac{1}{3}$$

$$x = k \cdot 4 \vee x = k \cdot 1\frac{1}{3}$$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot 1\frac{1}{3}$.

63

a $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$

$$2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{7}{12}\pi \vee x = \frac{13}{36}\pi \vee x = 1\frac{1}{36}\pi \vee x = 1\frac{25}{36}\pi$$

b $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$

$$3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi \vee 3x + \frac{1}{2}\pi = -(2x - \frac{1}{4}\pi) + k \cdot 2\pi$$

$$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$$

$$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 5x = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{4}\pi \vee x = \frac{7}{20}\pi \vee x = \frac{3}{4}\pi \vee x = 1\frac{3}{20}\pi \vee x = 1\frac{11}{20}\pi \vee x = 1\frac{19}{20}\pi$$

64 $f(x) = g(x)$ geeft $\sin(x - \frac{1}{4}\pi) = \sin(2x - \frac{1}{3}\pi)$

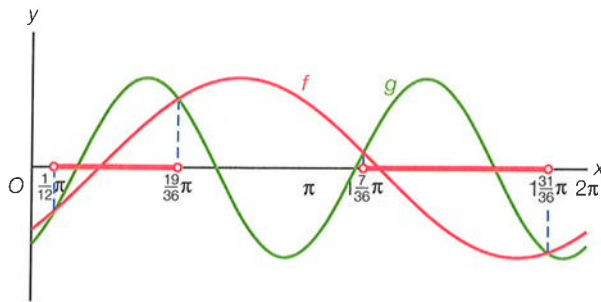
$$x - \frac{1}{4}\pi = 2x - \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \pi - (2x - \frac{1}{3}\pi) + k \cdot 2\pi$$

$$-x = -\frac{1}{12}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \pi - 2x + \frac{1}{3}\pi + k \cdot 2\pi$$

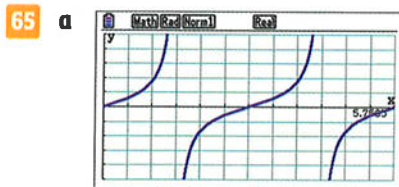
$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{7}{12}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee x = \frac{19}{36}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{12}\pi \vee x = \frac{19}{36}\pi \vee x = 1\frac{7}{36}\pi \vee x = 1\frac{31}{36}\pi$$



$$f(x) < g(x) \text{ geeft } \frac{1}{12}\pi < x < \frac{19}{36}\pi \vee 1\frac{7}{36}\pi < x < 1\frac{31}{36}\pi$$



b Dat zijn de lijnen $x = \frac{1}{2}\pi$ en $x = 1\frac{1}{2}\pi$.

66 a $\left. \begin{array}{l} \tan(A) = \frac{\sin(A)}{\cos(A)} \\ \tan(A) = 0 \end{array} \right\} \begin{array}{l} \sin(A) = 0 \wedge \cos(A) \neq 0 \\ A = k \cdot \pi \end{array}$

b $\tan(2x - \frac{1}{6}\pi) = 0$

$$2x - \frac{1}{6}\pi = k \cdot \pi$$

$$2x = \frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$$

Bladzijde 161

67 a $\tan(3x - \frac{1}{6}\pi) = \frac{1}{3}\sqrt{3}$

$$3x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot \pi$$

$$3x = \frac{1}{3}\pi + k \cdot \pi$$

$$x = \frac{1}{9}\pi + k \cdot \frac{1}{3}\pi$$

b $1 - \tan(\frac{3}{4}x) = 2$

$$\tan(\frac{3}{4}x) = -1$$

$$\frac{3}{4}x = \frac{3}{4}\pi + k \cdot \pi$$

$$x = \pi + k \cdot 1\frac{1}{3}\pi$$

c $\tan(\frac{1}{2}x) = \tan(2x - \frac{1}{6}\pi)$

$$\frac{1}{2}x = 2x - \frac{1}{6}\pi + k \cdot \pi$$

$$-\frac{1}{2}x = -\frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

d $3 \tan(\frac{1}{2}\pi x) = \sqrt{3}$

$$\tan(\frac{1}{2}\pi x) = \frac{1}{3}\sqrt{3}$$

$$\frac{1}{2}\pi x = \frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{3} + k \cdot 2$$

e $2 + \sqrt{3} \cdot \tan(\frac{1}{8}\pi x) = 5$

$$\sqrt{3} \cdot \tan(\frac{1}{8}\pi x) = 3$$

$$\tan(\frac{1}{8}\pi x) = \sqrt{3}$$

$$\frac{1}{8}\pi x = \frac{1}{3}\pi + k \cdot \pi$$

$$x = 2\frac{2}{3} + k \cdot 8$$

f $\tan(\frac{1}{3}\pi x) = \tan(\frac{1}{4}\pi(x-1))$

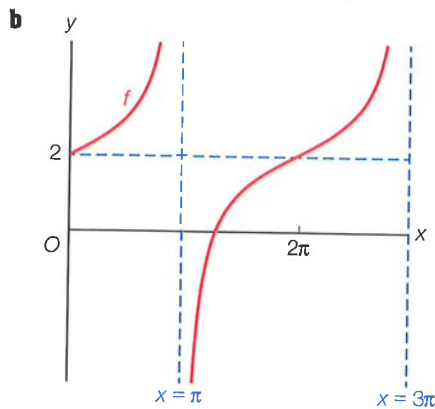
$$\frac{1}{3}\pi x = \frac{1}{4}\pi(x-1) + k \cdot \pi$$

$$\frac{1}{3}\pi x = \frac{1}{4}\pi x - \frac{1}{4}\pi + k \cdot \pi$$

$$\frac{1}{12}\pi x = -\frac{1}{4}\pi + k \cdot \pi$$

$$x = -3 + k \cdot 12$$

- 68 a** Beginpunt $(0, 2)$, periode $\frac{\pi}{\frac{1}{2}} = 2\pi$ en asymptoten zijn de lijnen $x = \pi$ en $x = 3\pi$.



- c** Voer in $y_1 = 2 + \tan(\frac{1}{2}x)$.
De optie nulpunt geeft $x \approx 4,07$.
Dus het nulpunt is 4,07.

- d** $f(x) = 3$ geeft $2 + \tan(\frac{1}{2}x) = 3$

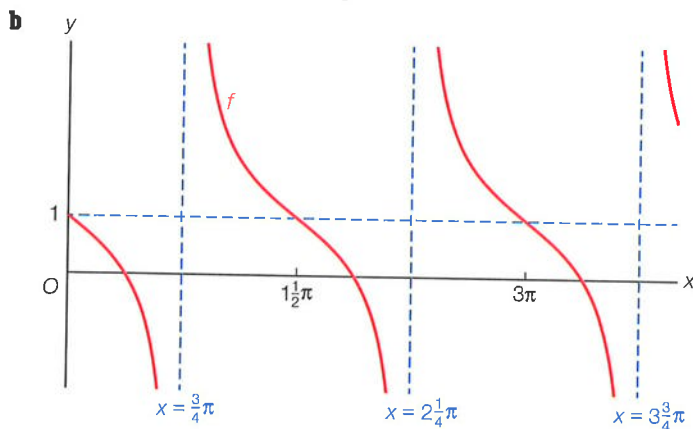
$$\tan(\frac{1}{2}x) = 1$$

$$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \pi$$

$$x = \frac{1}{2}\pi + k \cdot 2\pi$$

x in $[0, 3\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 2\frac{1}{2}\pi$, dus de snijpunten zijn $(\frac{1}{2}\pi, 3)$ en $(2\frac{1}{2}\pi, 3)$.

- 69 a** Beginpunt $(0, 1)$, periode $\frac{\pi}{\frac{2}{3}} = 1\frac{1}{2}\pi$ en asymptoten zijn de lijnen $x = \frac{3}{4}\pi$, $x = 2\frac{1}{4}\pi$ en $x = 3\frac{3}{4}\pi$.



- c** $f(x) = 0$ geeft $1 - \tan(\frac{2}{3}x) = 0$

$$\tan(\frac{2}{3}x) = 1$$

$$\frac{2}{3}x = \frac{1}{4}\pi + k \cdot \pi$$

$$x = \frac{3}{8}\pi + k \cdot 1\frac{1}{2}\pi$$

x in $[0, 4\pi]$ geeft $x = \frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi \vee x = 3\frac{3}{8}\pi$

Dus de nulpunten zijn $\frac{3}{8}\pi$, $1\frac{7}{8}\pi$ en $3\frac{3}{8}\pi$.

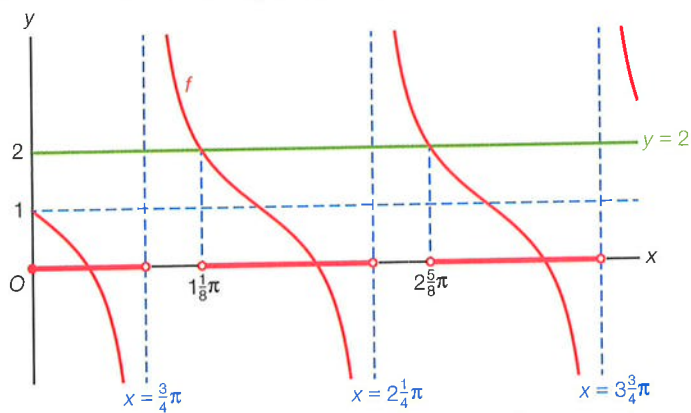
d $f(x) = 2$ geeft $1 - \tan(\frac{2}{3}x) = 2$

$$\tan(\frac{2}{3}x) = -1$$

$$\frac{2}{3}x = \frac{3}{4}\pi + k \cdot \pi$$

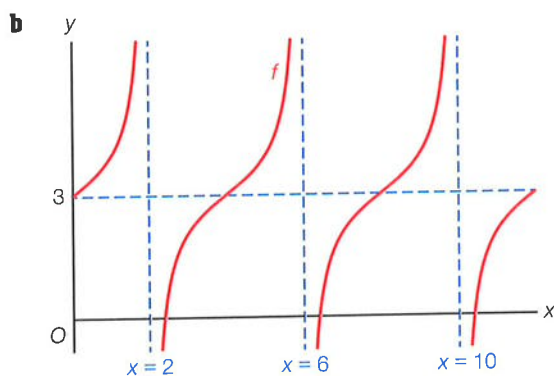
$$x = 1\frac{1}{8}\pi + k \cdot 1\frac{1}{2}\pi$$

x in $[0, 4\pi]$ geeft $x = 1\frac{1}{8}\pi \vee x = 2\frac{5}{8}\pi$



$f(x) < 2$ geeft $0 \leq x < \frac{3}{4}\pi \vee 1\frac{1}{8}\pi < x < 2\frac{1}{4}\pi \vee 2\frac{5}{8}\pi < x < 3\frac{3}{4}\pi$

70 a Beginpunt $(0, 3)$, periode $\frac{\pi}{\frac{1}{4}\pi} = 4$ en asymptoten zijn de lijnen $x = 2$, $x = 6$ en $x = 10$.



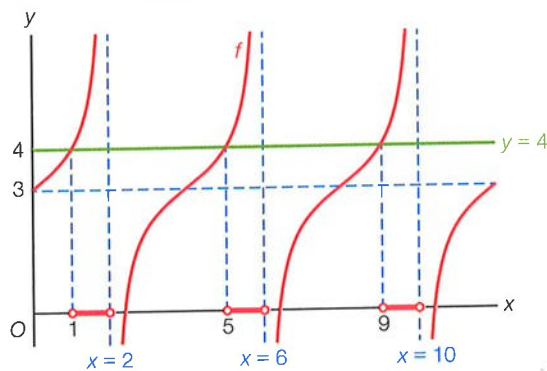
c $f(x) = 4$ geeft $3 + \tan(\frac{1}{4}\pi x) = 4$

$$\tan(\frac{1}{4}\pi x) = 1$$

$$\frac{1}{4}\pi x = \frac{1}{4}\pi + k \cdot \pi$$

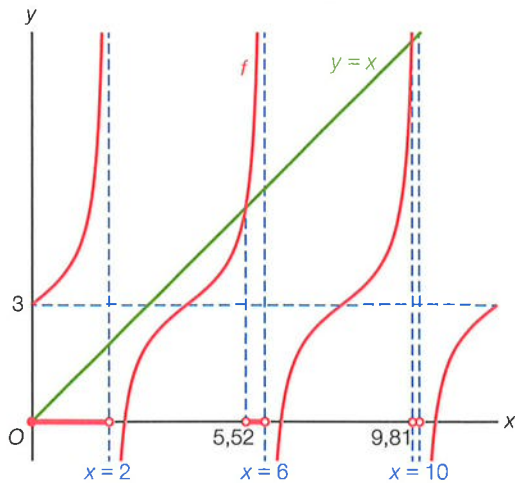
$$x = 1 + k \cdot 4$$

x in $[0, 12]$ geeft $x = 1 \vee x = 5 \vee x = 9$



$f(x) > 4$ geeft $1 < x < 2 \vee 5 < x < 6 \vee 9 < x < 10$

- d Voer in $y_1 = 3 + \tan(\frac{1}{4}\pi x)$ en $y_2 = x$.
De optie snijpunt geeft $x \approx 5,52$ en $x \approx 9,81$.



$f(x) > x$ geeft $0 \leq x < 2 \vee 5,52 < x < 6 \vee 9,81 < x < 10$

71 $\sin(2x - \frac{1}{3}\pi) = -\cos(2x - \frac{1}{3}\pi)$
 $\frac{\sin(2x - \frac{1}{3}\pi)}{\cos(2x - \frac{1}{3}\pi)} = -1$
 $\tan(2x - \frac{1}{3}\pi) = -1$
 $2x - \frac{1}{3}\pi = \frac{3}{4}\pi + k \cdot \pi$
 $2x = 1\frac{1}{12}\pi + k \cdot \pi$
 $x = \frac{13}{24}\pi + k \cdot \frac{1}{2}\pi$

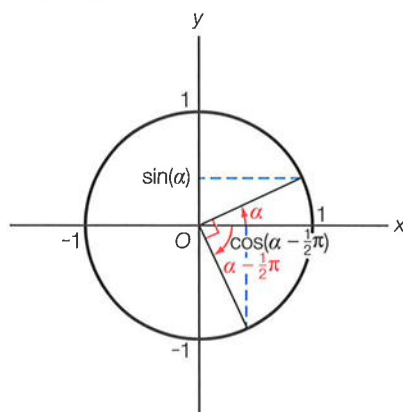
8.4 Herleiden en differentiëren

Bladzijde 163

72 $\sin(-\alpha) = y_Q = -y_P = -\sin(\alpha)$
 $\cos(-\alpha) = x_Q = x_P = \cos(\alpha)$

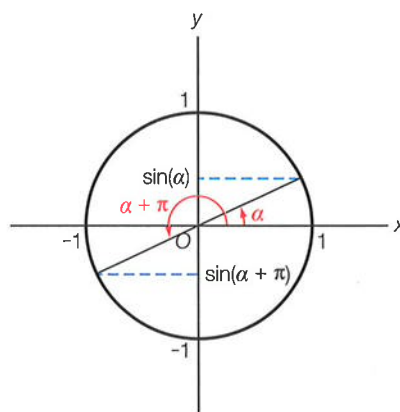
Bladzijde 164

73 a



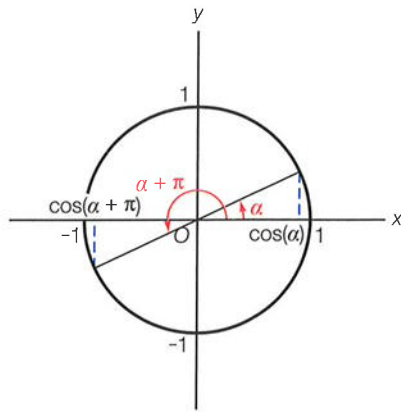
$$\cos(\alpha - \frac{1}{2}\pi) = \sin(\alpha)$$

b



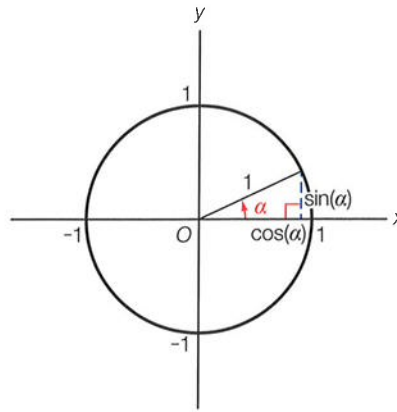
$$\sin(\alpha + \pi) = -\sin(\alpha)$$

c



$$\cos(\alpha + \pi) = -\cos(\alpha)$$

d



De stelling van Pythagoras geeft $\sin^2(\alpha) + \cos^2(\alpha) = 1$.

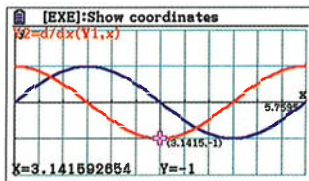
- 74 a** $\sin(x + \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi - \frac{1}{2}\pi) = \cos(x - \frac{1}{3}\pi)$
b $\cos(2x + \frac{1}{3}\pi) = \sin(2x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + \frac{5}{6}\pi)$
c $-\sin(3x - \frac{2}{3}\pi) = \sin(3x - \frac{2}{3}\pi + \pi) = \sin(3x + \frac{1}{3}\pi) = \cos(3x + \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(3x - \frac{1}{6}\pi)$
d $-\cos(4x + \frac{1}{6}\pi) = \cos(4x + \frac{1}{6}\pi + \pi) = \cos(4x + 2\frac{1}{6}\pi) = \sin(4x + 2\frac{1}{6}\pi + \frac{1}{2}\pi) = \sin(4x + 2\frac{2}{3}\pi) = \sin(4x + \frac{2}{3}\pi)$

75 a $(\sin(x) - \cos(x))^2 = \sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x) = \sin^2(x) + \cos^2(x) - 2\sin(x)\cos(x) = 1 - 2\sin(x)\cos(x)$

b $\frac{2\sin^2(x) + \cos^2(x)}{\cos^2(x)} = \frac{2\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = 2\left(\frac{\sin(x)}{\cos(x)}\right)^2 + 1 = 2\tan^2(x) + 1$

- 76 a** $\sin^2(x) + 4\cos(x) = 1 - \cos^2(x) + 4\cos(x)$
b $2\cos^2(x) + \sin(x) - 2 = 2(1 - \sin^2(x)) + \sin(x) - 2 = 2 - 2\sin^2(x) + \sin(x) - 2 = -2\sin^2(x) + \sin(x)$
c $2\sin^2(x) + \cos^2(x) + \cos(x) = 2(1 - \cos^2(x)) + \cos^2(x) + \cos(x) = 2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = -\cos^2(x) + \cos(x) + 2$

- 77 a** Voer in $y_1 = \sin(x)$ en $y_2 = \frac{d}{dx}(y_1) \Big|_{x=x}$.



- b** Waarschijnlijk is $y = \cos(x)$ de afgeleide van $y = \sin(x)$.

NORM DRIJF AUTO REZEL RAD WIN			
DRUK OP ENTER VOOR BELIEKEN			
X	Y1	Y2	
0	1	0.5403	
1	0.8415	-0.416	
2	-0.911	-0.99	
3	-0.1415	-0.654	
4	0.2037	0.2037	
5	0.9602	0.9602	
6	0.7539	0.7539	
7	-0.1415	-0.1415	
8	-0.911	-0.911	
9	-0.839	-0.839	
10			

- c** Voer in $y_1 = \cos(x)$ en $y_2 = \frac{d}{dx}(y_1) \Big|_{x=x}$.



Waarschijnlijk is $y = -\sin(x)$ de afgeleide van $y = \cos(x)$.

NORM DRIJF AUTO REEEL RAD HN DRUK OP ENTER VOOR BEWERKEN				
X	Y1	Y2		
0	0	0		
1	-0.841	-0.841		
2	-0.909	-0.909		
3	-0.141	-0.141		
4	0.7568	0.7568		
5	0.9589	0.9589		
6	0.2794	0.2794		
7	-0.657	-0.657		
8	-0.989	-0.989		
9	-0.412	-0.412		
10	0.544	0.544		
Y1 = -sin(X)				

- d $g(x) = \cos(x) = \sin(x + \frac{1}{2}\pi)$ geeft $g'(x) = \cos(x + \frac{1}{2}\pi) = \sin(x + \frac{1}{2}\pi + \frac{1}{2}\pi) = \sin(x + \pi) = -\sin(x)$
Dus $g(x) = \cos(x)$ geeft $g'(x) = -\sin(x)$.

Bladzijde 166

- 78 a $f(x) = 3 + 4 \sin(2x - \frac{1}{3}\pi)$ geeft $f'(x) = 4 \cos(2x - \frac{1}{3}\pi) \cdot 2 = 8 \cos(2x - \frac{1}{3}\pi)$
b $g(x) = x \cos(x)$ geeft $g'(x) = 1 \cdot \cos(x) + x \cdot -\sin(x) = \cos(x) - x \sin(x)$
c $h(x) = \frac{x^2 + \sin(x)}{\cos(x)}$ geeft $h'(x) = \frac{\cos(x) \cdot (2x + \cos(x)) - (x^2 + \sin(x)) \cdot -\sin(x)}{\cos^2(x)}$
$$= \frac{2x \cos(x) + \cos^2(x) + x^2 \sin(x) + \sin^2(x)}{\cos^2(x)} = \frac{2x \cos(x) + x^2 \sin(x) + 1}{\cos^2(x)}$$

d $k(x) = \cos^2(x)$ geeft $k'(x) = 2 \cos(x) \cdot -\sin(x) = -2 \sin(x) \cos(x)$
e $l(x) = x + 3 \sin^2(x)$ geeft $l'(x) = 1 + 6 \sin(x) \cos(x)$
f $j(x) = \frac{\cos(x)}{\sin(x)}$ geeft $j'(x) = \frac{\sin(x) \cdot -\sin(x) - \cos(x) \cdot \cos(x)}{\sin^2(x)} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)}$
- 79 a $f(x) = 10 + 16 \cos(\frac{1}{2}(x-1))$ geeft $f'(x) = -16 \sin(\frac{1}{2}(x-1)) \cdot \frac{1}{2} = -8 \sin(\frac{1}{2}(x-1))$
b $g(x) = x^2 \sin(3x)$ geeft $g'(x) = 2x \cdot \sin(3x) + x^2 \cdot \cos(3x) \cdot 3 = 2x \sin(3x) + 3x^2 \cos(3x)$
c $h(x) = \frac{\sin(x)}{x^2 + \cos(x)}$ geeft $h'(x) = \frac{(x^2 + \cos(x)) \cdot \cos(x) - \sin(x) \cdot (2x - \sin(x))}{(x^2 + \cos(x))^2}$
$$= \frac{x^2 \cos(x) + \cos^2(x) - 2x \sin(x) + \sin^2(x)}{(x^2 + \cos(x))^2} = \frac{x^2 \cos(x) - 2x \sin(x) + 1}{(x^2 + \cos(x))^2}$$

d $j(x) = 2 \sin^2(x)$ geeft $j'(x) = 4 \sin(x) \cos(x)$
- 80 a $f(x) = x \cos(2x)$ geeft $f'(x) = 1 \cdot \cos(2x) + x \cdot -\sin(2x) \cdot 2 = \cos(2x) - 2x \sin(2x)$
b $g(x) = 2x \sin(3x-1)$ geeft $g'(x) = 2 \cdot \sin(3x-1) + 2x \cdot \cos(3x-1) \cdot 3 = 2 \sin(3x-1) + 6x \cos(3x-1)$
c $h(x) = \frac{x \sin(x)}{x + \sin(x)}$ geeft $h'(x) = \frac{(x + \sin(x)) \cdot (1 \cdot \sin(x) + x \cdot \cos(x)) - x \sin(x) \cdot (1 + \cos(x))}{(x + \sin(x))^2}$
$$= \frac{x \sin(x) + x^2 \cos(x) + \sin^2(x) + x \sin(x) \cos(x) - x \sin(x) - x \sin(x) \cos(x)}{(x + \sin(x))^2}$$

$$= \frac{x^2 \cos(x) + \sin^2(x)}{(x + \sin(x))^2}$$

d $j(x) = 1 + 2 \cos^2(x)$ geeft $j'(x) = 4 \cos(x) \cdot -\sin(x) = -4 \sin(x) \cos(x)$

Bladzijde 167

- 81 a $f(x) = 3 \tan(2x)$ geeft $f'(x) = 3 \cdot \frac{1}{\cos^2(2x)} \cdot 2 = \frac{6}{\cos^2(2x)}$
b $g(x) = \tan^2(x)$ geeft $g'(x) = 2 \tan(x) \cdot \frac{1}{\cos^2(x)} = 2 \cdot \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos^2(x)} = \frac{2 \sin(x)}{\cos^3(x)}$

- 82** a $f(x) = \cos^3(x)$ geeft $f'(x) = 3 \cos^2(x) \cdot -\sin(x) = -3 \sin(x) \cos^2(x)$
 b $g(x) = \cos(x^3)$ geeft $g'(x) = -\sin(x^3) \cdot 3x^2 = -3x^2 \sin(x^3)$
 c $h(x) = \sqrt{\sin(x)}$ geeft $h'(x) = \frac{1}{2\sqrt{\sin(x)}} \cdot \cos(x) = \frac{\cos(x)}{2\sqrt{\sin(x)}}$
 d $j(x) = \cos^2(x) - \sin^2(x)$ geeft $j'(x) = 2 \cos(x) \cdot -\sin(x) - 2 \sin(x) \cdot \cos(x) = -4 \sin(x) \cos(x)$

- 83** a $f(x) = \sin^3(x) + \sin(x)$ geeft $f'(x) = 3 \sin^2(x) \cos(x) + \cos(x)$
 $= 3(1 - \cos^2(x)) \cos(x) + \cos(x)$
 $= 3 \cos(x) - 3 \cos^3(x) + \cos(x)$
 $= 4 \cos(x) - 3 \cos^3(x)$
 b $g(x) = \sin^2(x) \cos(x)$ geeft $g'(x) = 2 \sin(x) \cos(x) \cdot \cos(x) + \sin^2(x) \cdot -\sin(x)$
 $= 2 \sin(x) \cos^2(x) - \sin^3(x)$
 $= 2 \sin(x) (1 - \sin^2(x)) - \sin^3(x)$
 $= 2 \sin(x) - 2 \sin^3(x) - \sin^3(x)$
 $= 2 \sin(x) - 3 \sin^3(x)$
 c $h(x) = \frac{\tan(x)}{\sin(x)} = \frac{\left(\frac{\sin(x)}{\cos(x)}\right)}{\sin(x)} = \frac{1}{\cos(x)}$ geeft $h'(x) = \frac{\cos(x) \cdot 0 - 1 \cdot -\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)}$

- 84** a $f(x) = \frac{3 \cos(x)}{2 - \sin(x)}$ geeft $f'(x) = \frac{(2 - \sin(x)) \cdot -3 \sin(x) - 3 \cos(x) \cdot -\cos(x)}{(2 - \sin(x))^2}$
 $= \frac{-6 \sin(x) + 3 \sin^2(x) + 3 \cos^2(x)}{(2 - \sin(x))^2} = \frac{-6 \sin(x) + 3}{(2 - \sin(x))^2}$

- b $f(x) = 0$ geeft $\frac{3 \cos(x)}{2 - \sin(x)} = 0$
 $\cos(x) = 0$
 $x = \frac{1}{2}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi$

Dus $A(\frac{1}{2}\pi, 0)$.

Stel $k: y = ax + b$ met $a = f'(\frac{1}{2}\pi) = \frac{-6 \cdot 1 + 3}{(2 - 1)^2} = \frac{-3}{1} = -3$.

$$\left. \begin{array}{l} y = -3x + b \\ \text{door } A(\frac{1}{2}\pi, 0) \end{array} \right\} \begin{array}{l} -3 \cdot \frac{1}{2}\pi + b = 0 \\ b = \frac{1}{2}\pi \end{array}$$

Dus $k: y = -3x + \frac{1}{2}\pi$.

- c $f'(x) = 0$ geeft $\frac{-6 \sin(x) + 3}{(2 - \sin(x))^2} = 0$
 $-6 \sin(x) + 3 = 0$
 $\sin(x) = \frac{1}{2}$
 $x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$

$$\text{max. is } f(\frac{1}{6}\pi) = \frac{3 \cos(\frac{1}{6}\pi)}{2 - \sin(\frac{1}{6}\pi)} = \frac{3 \cdot \frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \sqrt{3}$$

$$\text{min. is } f(\frac{5}{6}\pi) = \frac{3 \cos(\frac{5}{6}\pi)}{2 - \sin(\frac{5}{6}\pi)} = \frac{3 \cdot -\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = -\sqrt{3}$$

Dus $B_f = [-\sqrt{3}, \sqrt{3}]$.

85 $f_p(x) = \cos^2(x) + \cos(x) + p$ geeft $f'_p(x) = 2 \cos(x) \cdot -\sin(x) - \sin(x) = -2 \sin(x) \cos(x) - \sin(x)$

$f'_p(x) = 0$ geeft $-2 \sin(x) \cos(x) - \sin(x) = 0$

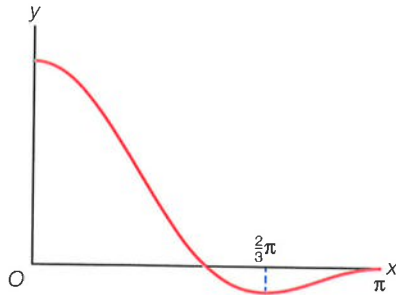
$-\sin(x)(2 \cos(x) + 1) = 0$

$\sin(x) = 0 \vee \cos(x) = -\frac{1}{2}$

$x = k \cdot \pi \vee x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$

x in $[0, \pi]$ geeft $x = 0 \vee x = \pi \vee x = \frac{2}{3}\pi$

Hieronder zie je een schets van de grafiek van $y = \cos^2(x) + \cos(x)$.



De grafiek van $y = \cos^2(x) + \cos(x)$ heeft een minimum voor $x = \frac{2}{3}\pi$, dus ook de grafiek van $f_p(x) = \cos^2(x) + \cos(x) + p$ heeft een minimum voor $x = \frac{2}{3}\pi$.

$f_p(\frac{2}{3}\pi) = 1$ geeft $\cos^2(\frac{2}{3}\pi) + \cos(\frac{2}{3}\pi) + p = 1$

$(-\frac{1}{2})^2 - \frac{1}{2} + p = 1$

$\frac{1}{4} - \frac{1}{2} + p = 1$

$p = 1\frac{1}{4}$

Diagnostische toets

Bladzijde 170

1 $A(\cos(40^\circ), \sin(40^\circ)) \approx A(0,77; 0,64)$

De draaiingshoek die bij het punt B hoort is $40^\circ + 120^\circ = 160^\circ$.

$B(\cos(160^\circ), \sin(160^\circ)) \approx B(-0,94; 0,34)$

De draaiingshoek die bij het punt C hoort is $40^\circ + 240^\circ = 280^\circ$.

$C(\cos(280^\circ), \sin(280^\circ)) \approx C(0,17; -0,98)$

2 Mogelijkheid 1, zie de figuur hiernaast.

$x_P = -0,33$, dus $\cos(\alpha_P) = -0,33$.

De GR geeft $1,907\dots$

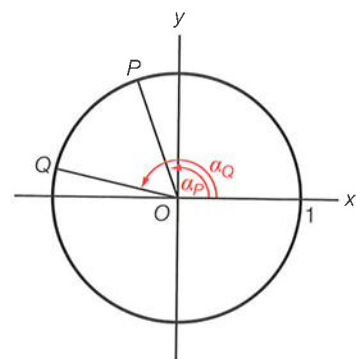
Dus $\alpha_P = 1,907\dots$

$y_Q = 0,22$, dus $\sin(\alpha_Q) = 0,22$.

De GR geeft $0,221\dots$

Dus $\alpha_Q = \pi - 0,221\dots = 2,919\dots$

Dus $\angle POQ = 2,919\dots - 1,907\dots \approx 1,01$.

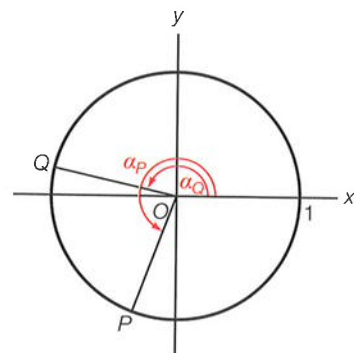


Mogelijkheid 2, zie de figuur hiernaast.

$\alpha_P = 2\pi - 1,907\dots = 4,376\dots$

$\alpha_Q = 2,919\dots$

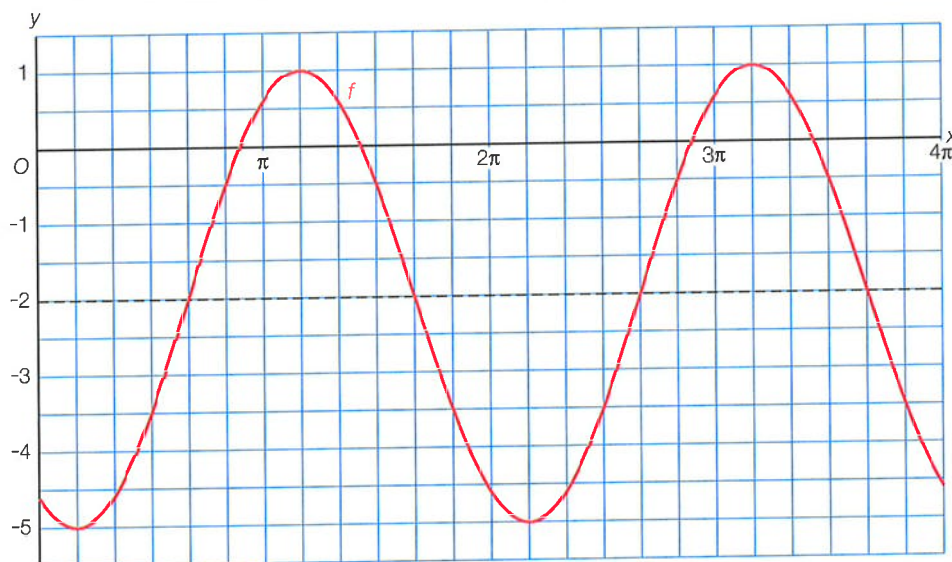
Dus $\angle POQ = 4,376\dots - 2,919\dots \approx 1,46$.



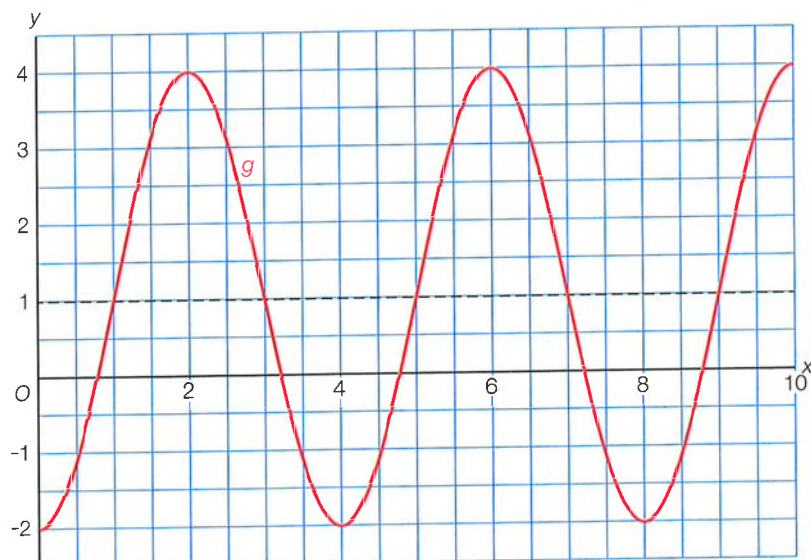
- 3 a** $\sin(x) = -\frac{1}{2}$ met $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$ geeft $x = -\frac{1}{6}\pi$
b $x = \cos(\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$
c $\cos(x) = \frac{1}{2}$ en $\frac{1}{2}\pi \leq x \leq 2\pi$ geeft $x = \frac{5}{3}\pi$
d $x = \sin(\frac{2}{3}\pi) = \frac{\sqrt{3}}{2}$
e $\sin(x) = \frac{1}{2}\sqrt{3}$ en $\frac{1}{2}\pi \leq x \leq \frac{3}{2}\pi$ geeft $x = \frac{5}{6}\pi$
f $x = \cos(\frac{2}{3}\pi) = -\frac{1}{2}$

- 4 a** De evenwichtsstand is -4 , de amplitude is 3 , de periode is 2π en een beginpunt is $(-\frac{2}{3}\pi, -4)$.
b De evenwichtsstand is $3,4$, de amplitude is 4 , de periode is $\frac{2}{5}\pi$ en een beginpunt is $(0,4; -0,6)$.

- 5 a** $f(x) = -2 - 3\sin(x + \frac{1}{3}\pi)$
 evenwichtsstand -2
 amplitude 3
 periode 2π
 $-3 < 0$, dus de grafiek gaat dalend door het punt $(\frac{2}{3}\pi, -2)$.



- b** $g(x) = 1 + 3\cos(\frac{1}{2}\pi x - \pi) = 1 + 3\cos(\frac{1}{2}\pi(x - 2))$
 evenwichtsstand 1
 amplitude 3
 periode $\frac{2\pi}{\frac{1}{2}\pi} = 4$
 $3 > 0$, dus $(2, 4)$ is een hoogste punt van de grafiek.



- 6 a** $a = \frac{10 + -30}{2} = -10$
 $b = 10 - -10 = 20$
 $c = \frac{2\pi}{30} = \frac{1}{15}\pi$
 $y = a + b \sin(c(x - d))$ met $b > 0$
 Stijgend door de evenwichtsstand bij $x = 10$, dus $d = 10$.
 $y = -10 + 20 \sin(\frac{1}{15}\pi(x - 10))$
- b** $y = a + b \cos(c(x - d))$ met $b > 0$
 Hoogste punt is $(17\frac{1}{2}, 10)$, dus $d = 17\frac{1}{2}$.
 $y = -10 + 20 \cos(\frac{1}{15}\pi(x - 17\frac{1}{2}))$
- c** $y = a + b \sin(c(x - d))$ met $b < 0$
 Dalend door de evenwichtsstand bij $x = 25$, dus $d = 25$.
 $y = -10 - 20 \sin(\frac{1}{15}\pi(x - 25))$
- d** $y = a + b \cos(c(x - d))$ met $b < 0$
 Laagste punt is $(2\frac{1}{2}, -30)$, dus $d = 2\frac{1}{2}$.
 $y = -10 - 20 \cos(\frac{1}{15}\pi(x - 2\frac{1}{2}))$

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- 7 a** $\sin(2x - \frac{1}{2}\pi) = -1$
 $2x - \frac{1}{2}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$
 $2x = 2\pi + k \cdot 2\pi$
 $x = \pi + k \cdot \pi$
 $x = k \cdot \pi$
- b** $\cos(2x + \frac{1}{3}\pi) = 0$
 $2x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$
 $2x = \frac{1}{6}\pi + k \cdot \pi$
 $x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$
- c** $\cos(\frac{1}{2}\pi x) \cdot (\cos(2\pi x) + 1) = 0$
 $\cos(\frac{1}{2}\pi x) = 0 \vee \cos(2\pi x) = -1$
 $\frac{1}{2}\pi x = \frac{1}{2}\pi + k \cdot \pi \vee 2\pi x = \pi + k \cdot 2\pi$
 $x = \frac{1}{3} + k \cdot \frac{2}{3} \vee x = \frac{1}{2} + k \cdot 1$
- d** $\sin^3(\frac{1}{3}\pi x) + \sin^2(\frac{1}{3}\pi x) = 0$
 $\sin^2(\frac{1}{3}\pi x)(\sin(\frac{1}{3}\pi x) + 1) = 0$
 $\sin(\frac{1}{3}\pi x) = 0 \vee \sin(\frac{1}{3}\pi x) = -1$
 $\frac{1}{3}\pi x = k \cdot \pi \vee \frac{1}{3}\pi x = 1\frac{1}{2}\pi + k \cdot 2\pi$
 $x = k \cdot 3 \vee x = 4\frac{1}{2} + k \cdot 6$
- 8 a** $4 \sin(2x + \frac{1}{2}\pi) = 2\sqrt{2}$
 $\sin(2x + \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$
 $2x + \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \frac{3}{4}\pi + k \cdot 2\pi$
 $2x = -\frac{1}{4}\pi + k \cdot 2\pi \vee 2x = \frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{1}{8}\pi + k \cdot \pi \vee x = \frac{1}{8}\pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = \frac{7}{8}\pi \vee x = 1\frac{7}{8}\pi \vee x = \frac{1}{8}\pi \vee x = 1\frac{1}{8}\pi$
- b** $\cos(x - \frac{1}{6}\pi) = -\frac{1}{2}\sqrt{3}$
 $x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$
 $x = \pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \pi \vee x = 1\frac{1}{3}\pi$

c $4 \cos^2(\frac{1}{2}x) = 2$

$$\cos^2(\frac{1}{2}x) = \frac{1}{2}$$

$$\cos(\frac{1}{2}x) = \frac{1}{2}\sqrt{2} \vee \cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{2}$$

$$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{3}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$$

$$x = \frac{1}{2}\pi + k \cdot 4\pi \vee x = -\frac{1}{2}\pi + k \cdot 4\pi \vee x = \frac{1}{2}\pi + k \cdot 4\pi \vee x = -\frac{1}{2}\pi + k \cdot 4\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = \frac{1}{2}\pi$$

d $\sin^2(\frac{3}{4}x + 1\frac{1}{2}\pi) = \frac{3}{4}$

$$\sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = \frac{1}{2}\sqrt{3} \vee \sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = -\frac{1}{2}\sqrt{3}$$

$$\frac{3}{4}x + 1\frac{1}{2}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = -\frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi$$

$$\frac{3}{4}x = -1\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -\frac{5}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -1\frac{5}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = -1\frac{5}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -1\frac{1}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -2\frac{4}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -\frac{2}{9}\pi + k \cdot 2\frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{5}{9}\pi \vee x = 1\frac{5}{9}\pi \vee x = \frac{2}{9}\pi$$

9 a $f(x) = -1$ geeft $-1 + 2 \cos(x - \frac{1}{3}\pi) = -1$

$$2 \cos(x - \frac{1}{3}\pi) = 0$$

$$\cos(x - \frac{1}{3}\pi) = 0$$

$$x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{5}{6}\pi + k \cdot \pi$$

$$x \text{ in } [-\pi, 3\pi] \text{ geeft } x = -\frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{5}{6}\pi \vee x = 2\frac{5}{6}\pi$$

Dus de grafiek van f snijdt de lijn van de evenwichtsstand in de punten

$$(-\frac{1}{6}\pi, -1), (\frac{5}{6}\pi, -1), (1\frac{5}{6}\pi, -1) \text{ en } (2\frac{5}{6}\pi, -1).$$

b $f(x) = 1$ geeft $-1 + 2 \cos(x - \frac{1}{3}\pi) = 1$

$$2 \cos(x - \frac{1}{3}\pi) = 2$$

$$\cos(x - \frac{1}{3}\pi) = 1$$

$$x - \frac{1}{3}\pi = k \cdot 2\pi$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x \text{ in } [-\pi, 3\pi] \text{ geeft } x = \frac{1}{3}\pi \vee x = 2\frac{1}{3}\pi$$

$$f(x) = -3 \text{ geeft } -1 + 2 \cos(x - \frac{1}{3}\pi) = -3$$

$$2 \cos(x - \frac{1}{3}\pi) = -2$$

$$\cos(x - \frac{1}{3}\pi) = -1$$

$$x - \frac{1}{3}\pi = \pi + k \cdot 2\pi$$

$$x = 1\frac{1}{3}\pi + k \cdot 2\pi$$

$$x \text{ in } [-\pi, 3\pi] \text{ geeft } x = -\frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi$$

De toppen van de grafiek van f zijn $(-\frac{2}{3}\pi, -3)$, $(\frac{1}{3}\pi, 1)$, $(1\frac{1}{3}\pi, -3)$ en $(2\frac{1}{3}\pi, 1)$.

c $f(x) = 0$ geeft $-1 + 2 \cos(x - \frac{1}{3}\pi) = 0$

$$2 \cos(x - \frac{1}{3}\pi) = 1$$

$$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$$

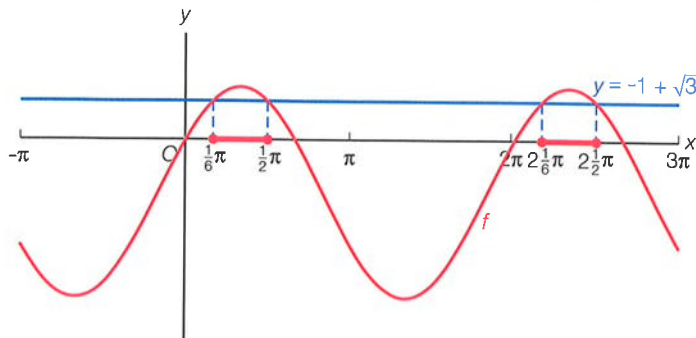
$$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi$$

$$x \text{ in } [-\pi, 3\pi] \text{ geeft } x = \frac{2}{3}\pi \vee x = 2\frac{2}{3}\pi \vee x = 0 \vee x = 2\pi$$

Dus de afstand tussen B en C is $2\pi - \frac{2}{3}\pi = 1\frac{1}{3}\pi$.

d $f(x) = -1 + \sqrt{3}$ geeft $-1 + 2 \cos(x - \frac{1}{3}\pi) = -1 + \sqrt{3}$
 $2 \cos(x - \frac{1}{3}\pi) = \sqrt{3}$
 $\cos(x - \frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$
 $x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi$
 x in $[-\pi, 3\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 2\frac{1}{2}\pi \vee x = \frac{1}{6}\pi \vee x = 2\frac{1}{6}\pi$



$f(x) \geq -1 + \sqrt{3}$ geeft $\frac{1}{6}\pi \leq x \leq \frac{1}{2}\pi \vee 2\frac{1}{6}\pi \leq x \leq 2\frac{1}{2}\pi$

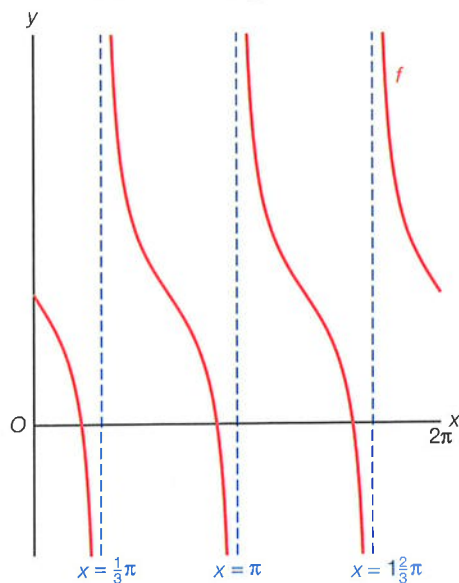
10 a $\sin(2x + \frac{1}{2}\pi) = \sin(x - \frac{1}{3}\pi)$
 $2x + \frac{1}{2}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \vee 3x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$
b $\cos(x - \frac{1}{6}\pi) = \cos(4x + \frac{1}{4}\pi)$
 $x - \frac{1}{6}\pi = 4x + \frac{1}{4}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -(4x + \frac{1}{4}\pi) + k \cdot 2\pi$
 $-3x = \frac{5}{12}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -4x - \frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{5}{36}\pi + k \cdot \frac{2}{3}\pi \vee 5x = -\frac{1}{12}\pi + k \cdot 2\pi$
 $x = -\frac{5}{36}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{60}\pi + k \cdot \frac{2}{5}\pi$
c $\tan(\frac{2}{3}\pi x) = -\frac{1}{3}\sqrt{3}$
 $\frac{2}{3}\pi x = \frac{5}{6}\pi + k \cdot \pi$
 $x = 1\frac{1}{4} + k \cdot 1\frac{1}{2}$
d $\tan(2x - \frac{1}{3}\pi) = \tan(x + \frac{1}{2}\pi)$
 $2x - \frac{1}{3}\pi = x + \frac{1}{2}\pi + k \cdot \pi$
 $x = \frac{5}{6}\pi + k \cdot \pi$

11 a $f(x) = 2 - \tan(1\frac{1}{2}x)$
 Een beginpunt is $(0, 2)$.

De periode is $\frac{\pi}{1\frac{1}{2}} = \frac{2}{3}\pi$.

De asymptoten zijn de lijnen $x = \frac{1}{3}\pi$, $x = \pi$ en $x = 1\frac{2}{3}\pi$.

- b** Voer in $y_1 = 2 - \tan(1\frac{1}{2}x)$.



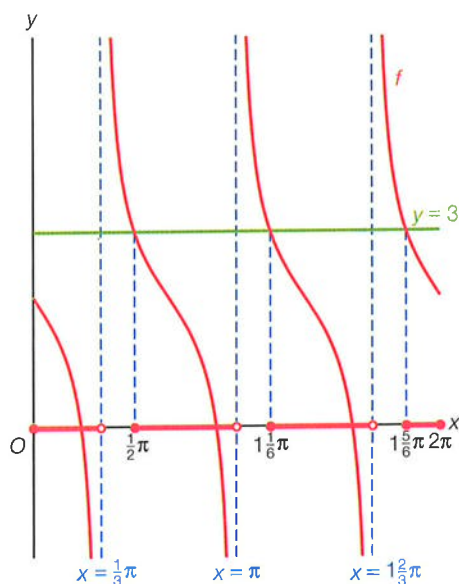
- c** $f(x) = 3$ geeft $2 - \tan(1\frac{1}{2}x) = 3$

$$\tan(1\frac{1}{2}x) = -1$$

$$1\frac{1}{2}x = \frac{3}{4}\pi + k \cdot \pi$$

$$x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi$$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$



$$f(x) \leq 3 \text{ geeft } 0 \leq x < \frac{1}{3}\pi \vee \frac{1}{2}\pi \leq x < \pi \vee 1\frac{1}{6}\pi \leq x < 1\frac{2}{3}\pi \vee 1\frac{5}{6}\pi \leq x \leq 2\pi$$

- 12 a** $\sin(2x - \frac{2}{3}\pi) = \cos(2x - \frac{2}{3}\pi - \frac{1}{2}\pi) = \cos(2x - 1\frac{1}{6}\pi)$
b $-\cos(5x + 1\frac{1}{4}\pi) = \cos(5x + 1\frac{1}{4}\pi + \pi) = \cos(5x + 2\frac{1}{4}\pi) = \sin(5x + 2\frac{1}{4}\pi + \frac{1}{2}\pi) = \sin(5x + 2\frac{3}{4}\pi) = \sin(5x + \frac{3}{4}\pi)$
c $(\sin(x) - \cos(x)) \cdot (\sin(x) + \cos(x)) \cdot \cos(x) = (\sin^2(x) - \cos^2(x)) \cdot \cos(x) =$
 $(1 - \cos^2(x) - \cos^2(x)) \cdot \cos(x) = (1 - 2\cos^2(x)) \cdot \cos(x) = \cos(x) - 2\cos^3(x)$

13 a $f(x) = x^3 \sin(4x)$ geeft $f'(x) = 3x^2 \cdot \sin(4x) + x^3 \cdot \cos(4x) \cdot 4 = 3x^2 \sin(4x) + 4x^3 \cos(4x)$

b $g(x) = \frac{\sin(2x)}{2 + \cos(x)}$ geeft

$$g'(x) = \frac{(2 + \cos(x)) \cdot \cos(2x) \cdot 2 - \sin(2x) \cdot -\sin(x)}{(2 + \cos(x))^2} = \frac{4 \cos(2x) + 2 \cos(x) \cos(2x) + \sin(x) \sin(2x)}{(2 + \cos(x))^2}$$

c $h(x) = 5 \sin^2(x)$ geeft $h'(x) = 10 \sin(x) \cos(x)$

d $j(x) = \tan(x^3)$ geeft $j'(x) = \frac{1}{\cos^2(x^3)} \cdot 3x^2 = \frac{3x^2}{\cos^2(x^3)}$

14 $f(x) = \frac{\sin(2x)}{2 + \sin(x)}$ geeft

$$f'(x) = \frac{(2 + \sin(x)) \cdot \cos(2x) \cdot 2 - \sin(2x) \cdot \cos(x)}{(2 + \sin(x))^2} = \frac{4 \cos(2x) + 2 \sin(x) \cos(2x) - \sin(2x) \cos(x)}{(2 + \sin(x))^2}$$

Stel $k: y = ax + b$ met $a = f'(\frac{1}{2}\pi) = \frac{4 \cos(\pi) + 2 \sin(\frac{1}{2}\pi) \cos(\pi) - \sin(\pi) \cos(\frac{1}{2}\pi)}{(2 + \sin(\frac{1}{2}\pi))^2} = \frac{-4 - 2 - 0}{(2 + 1)^2} = -\frac{2}{3}$.

$$\left. \begin{array}{l} y = -\frac{2}{3}x + b \\ f(\frac{1}{2}\pi) = 0, \text{ dus } A(\frac{1}{2}\pi, 0) \end{array} \right\} \begin{array}{l} -\frac{2}{3} \cdot \frac{1}{2}\pi + b = 0 \\ b = \frac{1}{3}\pi \end{array}$$

Dus $k: y = -\frac{2}{3}x + \frac{1}{3}\pi$.